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Computation of fundamental time-eigenvalue of the neutron transport equation

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ABSTRACT

A modified $\alpha - k$ power iteration method is presented for the prediction of time-eigenvalue(α) of the neutron transport equation. By developing a direct relationship between K-eigenvalue and α-eigenvalue, a new formula is introduced to estimate the value of α. Compared with the conventional method, it is not required to provide the initial values of α for the modified method. Since it is always difficult to guess the suitable initial values, the modified method is more convenient for solving time-eigenvalue problems. Computational experiences show that the accuracy of the modified method is the same as the conventional method.

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1. Introduction

Although several eigenvalue problems can be defined for the quasi-stationary neutron transport equation, the two most common formulations are effective fission multiplication factor (k_{eff}) and α- or time-eigenvalues, which are basic to the subject of nuclear reactor physics. The physical meaning and the properties of these eigenvalues were discussed in a very rich literature, e.g. (Bell and Glasstone, 1970; Larsen and Zweifel, 1974; Lewis and Miller, 1993).

In the K-eigenvalue problem, the fission source is artificially multiplied by a factor $1/k$ so as to obtain a balance between production and loss of neutrons and thus a steady state. The eigenvalues k_i are all real and positive. The largest value k_1 is called the effective fission multiplication factor k_{eff}. The fluxes corresponding to the k_1 are positive everywhere, whereas the fluxes may be negative for other modes (Lewis and Miller, 1993, p. 46; Modak and Gupta, 2007). The power iteration (PI) method (Duderstadt and Hamilton, 1976) is commonly used to obtain the k_{eff}.

The α-eigenvalue problem is defined in a different way. Let $\phi = \phi(\vec{r}, E, \vec{\Omega}, t)$ denote neutron angular flux at point $\vec{r}$ in the energy E and the direction $\vec{\Omega}$ at time t. In the α-eigenvalue problem, the flux is assumed to have an exponential time-dependence in the form

$$\phi(\vec{r}, E, \vec{\Omega}, t) = \phi_\alpha(\vec{r}, E, \vec{\Omega}) \cdot e^{\alpha t} \tag{1}$$

Then, by insertion in time-dependent neutron transport equation, the eigenvalue equation can be obtained. The eigenvalue α appears in the form of a 1/v absorber. Like the K-eigenvalue problem, there are many possible eigenvalues α_i and corresponding eigenfunctions. Unfortunately, only a few general properties of the α-eigenvalue are known so far. The α_i need not be real and positive. They may be composed of a continuous spectrum and a discrete spectrum. In particular, it has been shown that under mild assumptions a dominant discrete eigenvalue α exists, which is real, larger than the real parts of all the other α, and whose associated eigenfunction is non-negative (Larsen and Zweifel, 1974; Zoia et al., 2014). Usually, the dominant discrete eigenvalue is called the fundamental eigenvalue.

The standard power iteration method cannot be used to compute the α-eigenvalue because the magnitude of fundamental α-eigenvalue is not the largest (Modak and Gupta, 2007). To find the fundamental α-eigenvalue, a $\alpha - k$ power iteration method has been developed in some recent papers, and adopted by many neutron transport codes (Briesmeiser, 2000; Ye et al., 2007; Zoia et al., 2014; Zoia et al., 2015). Although the power iteration method dominates the field of eigenvalue computation, other methods have emerged, for instance, implicitly restarted Arnoldi method (IRAM) (Lathouwers, 2003; Kópházi and Lathouwers, 2012) and nonlinear solution method (Fichtl and Warsa, 2013). Kópházi and Lathouwers (2012) refers to the fact that the IRAM is

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applicable only to subcritical systems. Fichtl and Warsa recast the α-eigenvalue problem as a nonlinear problem, and solve this nonlinear problem by nonlinear solvers.

In order to find the α-eigenvalue by the $\alpha - k$ power iteration, it is required to provide a initial value of α (see Section 2.2 for details). It is always difficult to guess the suitable initial value. Therefore, an attempt has been made here to design a new $\alpha - k$ power iteration method for obtaining the α-eigenvalue of the neutron transport equation. The initial value is no longer needed for the new method.

The remainder of this paper is organized in the following manner. Section 2 briefly describes the K- and α-eigenvalue equations, and the conventional methods. Section 3 details the new computation scheme developed by this work. Section 4 presents some numerical results for typical problems to demonstrate the validity and efficiency of the new method. Finally, Section 5 gives conclusions.

Since only “prompt” α-eigenvalue problem is considered in this paper, the delayed neutrons are neglected. So, in the following equations, the notation ν_p is adopted to denote the average number of prompt neutrons per fission.

2. Eigenvalue problems and conventional methods

2.1. Eigenvalue problems

The α-eigenvalue equation can be derived from the time-dependent neutron transport equation given below:

$$\frac{1}{v}\frac{\partial\phi}{\partial t} + \vec{\Omega}\cdot\nabla\phi + \Sigma_t\phi = \int_0^\infty dE' \int_{4\pi} d\Omega' \Sigma_s\left(\vec{r}, E' \to E, \vec{\Omega}'\cdot\vec{\Omega}\right)\phi\left(\vec{r}, E', \vec{\Omega}', t\right) + \frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f(\vec{r}, E')\int_{4\pi} d\Omega'\phi\left(\vec{r}, E', \vec{\Omega}', t\right) \tag{2}$$

here the well known notations are used. If Eq. (1) is substituted in Eq. (2), the time derivative term is replaced by $\alpha/v\phi$ and then the common exponential term $e^{\alpha t}$ is removed, leading to the time-independent α-eigenvalue equation:

$$\vec{\Omega}\cdot\nabla\phi_\alpha + \left(\Sigma_t + \frac{\alpha}{v}\right)\phi_\alpha = \int_0^\infty dE' \int_{4\pi} d\Omega' \Sigma_s\left(\vec{r}, E' \to E, \vec{\Omega}'\cdot\vec{\Omega}\right)\phi_\alpha\left(\vec{r}, E', \vec{\Omega}'\right) + \frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f(\vec{r}, E')\int_{4\pi} d\Omega'\phi_\alpha\left(\vec{r}, E', \vec{\Omega}'\right) \tag{3}$$

The K-eigenvalue problem is formulated by assuming that ν_p, the average number of neutrons per fission, can be adjusted to obtain a time-independent solution to Eq. (2). Hence the K-eigenvalue equation may be written as follows:

$$\vec{\Omega}\cdot\nabla\phi_k + \Sigma_t\phi_k = \int_0^\infty dE' \int_{4\pi} d\Omega' \Sigma_s\left(\vec{r}, E' \to E, \vec{\Omega}'\cdot\vec{\Omega}\right)\phi_k\left(\vec{r}, E', \vec{\Omega}'\right) + \frac{1}{k}\frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f(\vec{r}, E')\int_{4\pi} d\Omega'\phi_k\left(\vec{r}, E', \vec{\Omega}'\right) \tag{4}$$

Finally, the common boundary conditions for the two problems are formed as

$$\phi_x\left(\vec{r}, E, \vec{\Omega}\right) = 0, \quad \vec{r}\in\partial V_B, \quad \vec{\Omega}\cdot\vec{n} < 0 \tag{5}$$

$$\phi_x\left(\vec{r}, E, \vec{\Omega}\right) = \phi_x\left(\vec{r}, E, \vec{\Omega}'\right), \quad \vec{r}\in\partial V_R, \quad \vec{\Omega}\cdot\vec{n} < 0 \tag{6}$$

for the bare and reflected boundaries, respectively. Here $\vec{\Omega}'$ is the reflected direction from $\vec{\Omega}$, and the subscript x is equal to α or k.

2.2. Conventional methods

Since the K-eigenvalues are real positive and the fundamental K-eigenvalue is the greatest, it is invariably solved by the method of power iteration. This method is well known and can be found in a very rich literature (Bell and Glasstone, 1970; Lewis and Miller, 1993; Du and Zhang, 1988), so further details are not presented.

Although the standard power iteration method cannot be used to find the α-eigenvalue (Modak and Gupta, 2007), a $\alpha - k$ power iteration method has been developed. By introducing a fictitious parameter k dividing the fission term, α-eigenvalue Eq. (3) can be written as follows:

$$\begin{aligned}\vec{\Omega}\cdot\nabla\phi_\alpha + \left(\Sigma_t + \frac{\alpha}{v}\right)\phi_\alpha = &\int_0^\infty dE' \int_{4\pi} d\Omega' \Sigma_s\left(\vec{r}, E' \to E, \vec{\Omega}'\cdot\vec{\Omega}\right) \\ &\times\phi_\alpha\left(\vec{r}, E', \vec{\Omega}'\right) + \frac{1}{k}\frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f(\vec{r}, E') \\ &\times\int_{4\pi} d\Omega'\phi_\alpha\left(\vec{r}, E', \vec{\Omega}'\right)\end{aligned} \tag{7}$$

which becomes a standard k-eigenvalue equation, the parameter α being though unknown. The basic strategy is to seek then the value α for which $k = 1$ (Zoia et al., 2014).

So, the α-eigenvalue can be found by the iterative scheme as follows (Du and Zhang, 1988):

1. α_1 and α_2 which are two initial values of α, are guessed. Then k_1 and k_2 are found by the power iteration from Eq. (7), respectively.
2. Based on (k_{n-2},α_{n-2}) and (k_{n-1},α_{n-1}), the new estimate of α, α_n, is obtained by a linear extrapolation (8). Subsequently, k_n can be found from Eq. (7).

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