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Conservative conservation equations: Numerical approach and code-to-code benchmarks

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ABSTRACT

One of the principle features of RELAP5-based system thermal hydraulic codes is the use of a two-fluid, non-equilibrium, non-homogeneous, hydrodynamic model for the transient simulation of the two-phase system behavior. This model includes six governing equations to describe the mass, energy, and momentum of the two fluids. The current version of RELAP-5 is not a fully conservative code because it uses both non-conservative and conservative numerical approximation forms of conservation equations. The current version of RELAP5 versions have mass and energy errors during time advancements, either resulting in (a) automatic reduction of time steps used in the advancement of the equations and increased run times or (b) the growth of unacceptably large errors in the transient results. Therefore, fully conservative conservation equations and closure equations have recently been developed to address this problem. This article demonstrates the numerical approach to implement the developed fully conservative conservation equations into RELAP5 and the results of RELAP5 including developed conservative form of conservation equations. RELAP5 versions including conservative and non-conservative conservation equations are compared for various tests from a single pipe to a whole Pressurized Water Reactor (PWR) model.

1. Introduction

Nuclear reactor systems are complex, and require detailed analysis to evaluate reactor performance during normal operations as well as accident or transient conditions. Computer codes that are used to analyze these complex reactor systems are called system codes". System codes are used in the design and analysis of nuclear reactors. They can be used to evaluate steady-state performance of reactor systems, and are also used for transient analyses. The system codes can aid reactor system engineers in refueling evolutions, relicensing reports with regulatory agencies, and applications for reactor plant power uprating. RELAP5 is one of the commonly used nuclear safety codes used for nuclear safety licensing and analyzing nuclear power plant systems. RELAP5 includes six governor equations for two fields (liquid and vapor). The conservative and non-conservative methods are two numerical approaches used for the solution of governing equations. Both methods are numerical approximations and introduce some error. However, a feature of conservative form of approximation is preserving mass and energy in a system. The current RELAP5 numerical scheme has two steps to evaluate basic conservation equations. The first evaluation uses non-conservative numerical approximation and the truncation errors in the linearization procedure of non-conservative numerical approximation may produce mass and energy errors during the advancement. The second step of semi-implicit scheme is to use the intermediate time variables, which are result from the non-conservative forms (expanded forms), to re-compute vapor/liquid internal energies and non-condensable fraction in the conservative forms (unexpanded forms) of governing equations. Because of utilizing the conservative forms, current RELAP5 is considered by many as a 'conservative' code. However, it is not fully conservative due to the fact:

1) Final pressure value is obtained by using non-conservative forms. Moreover, void fraction and phasic densities are all a function of the pressure.
2) Unexpanded forms (the second step) are not used for 'one-phase to on-phase' and 'one-phase to two-phase' conditions.

Therefore, when the masses or energies in the system are summed at the beginning and exit of boundary conditions, the

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summed values can be different from summation of mass or energy of each volume in the system (NSAD, 2001). Switching to the consistent conservative form of numerical approach reduces the loss or gain of mass and energy.

Codes with the fully conservative forms of governing equations have an advantage over others (Mahaffy, 1993) and the current RELAP5 is not a fully conservative code. Therefore, this work focuses on developing, implementing, and solving a set of fully conservative mass and energy equations in RELAP5. Fu et al. (2014) presents five conservative approximations of mass and energy conservation equations alone with sixteen constitutive equations. This article introduces RELAP5 solution strategy for conservative form of conservation equations. The new solution method constructs one matrix for each system with conservation equations and constitutive equations. It then solves the matrix to obtain the changes in all new time variables simultaneously. The system matrix is composed of two sub-matrices including volume conservation equations (mass and energy) and junction conservation equations (momentum). Since conservative numerical approximation is applied only for mass and energy conservation equations, the article gives only the coefficients used to build volume mass-energy sub-matrix with conservative mass-energy conservation equations and their closure equations. Moreover, the preliminary tests show the comparison results between the fully conservative and non-conservative approaches in RELAP5 (Roth, Aydogan, 2014a, 2014b). The improved code is implemented into RELAP5/SCDAP mod 4.0 version. The code verification was done by using the US-NRC's Regulatory Guide 1.203 (Rg 1.203, 2005).

2. Motivation

The thermal hydraulic codes, although primarily intended for transient use, require a steady state capability to establish initial conditions. The numerical procedures for these codes cannot be obtained simply by setting the time derivatives to zero. Thus, a transient with suitable boundary conditions is run to obtain a steady state. To address the effects of mass error in thermal hydraulic codes, a testing PWR plant model can be set up such that the primary system is modeled as a completely closed, liquid filled system and a pressurizer or an equivalent device is not included. However, this simulation simply fails to reach a steady state because of a poor choice of boundary conditions. In the case of the failed steady state run, the initial conditions were set to the high pressures of an operating PWR plant, but as the transient proceeded, the pressures dropped steadily to lower pressure values. As the solution was advanced, the mass errors for the system were negative and thus liquid was lost from the system. With the almost incompressible liquid water, even a small loss of water leads to a dramatic loss in pressure. The problem can be mitigated by modeling a pressurizer. For example, in the RELAP5 simplified PWR system, the pressurizer is modeled with volumes and junctions for both the liquid portion and the vapor portion of the pressurizer plus other volumes and junctions for relief valves. For the steady state problem, only the liquid portion of the pressurizer is modeled and a time dependent volume is connected by a junction to the top of the liquid portion of the pressurizer. The conditions in the time dependent volume are set to saturated liquid water at the desired pressure. If the pressurizer pressure were too high, water will flow out of the pressurizer, allowing the pressure in the pressurizer to drop. If the pressurizer pressure were too low, the saturated liquid at the desired pressure will flow into the pressurizer. Thus, the boundary condition mitigated the mass error. To proceed with the transient run, the renodalization capability is used to replace time dependent junction with the vapor portion of the accumulator.

Therefore, the mass error can be significant and could preclude application of the code several situations. Fortunately, most modeling situations include boundary conditions that can correct the mass error effects. Even so, reducing mass error is a worthwhile effort since an estimate of the mass error is printed in a prominent place in the output and users tend to check this quantity when evaluating the simulation.

3. Solution strategies of non-conservative method and conservative method

Before introducing the solution strategy for the new conservative method, the semi-implicit advancement solution strategy of the non-conservative method in current RELAP5 is briefly described first.

The numerical approximation to the momentum equations results in two equations per junction, involving only the liquid and vapor velocities for the junction and the pressures for the two volumes connected by each junction. Using the 2 by 2 matrices from the two equations per junction, expressions for the liquid and vapor velocities in terms of the pressures from the connected volumes can be obtained. The numerical approximations for the other conservations equations result in five equations per volume derived from the partial differential equations plus thirteen equations from the algebraic relationships. Using algebraic substitutions, variables can be eliminated until only five equations per volume remain, involving volume pressure, vapor void fraction, liquid and vapor internal energies, and non-condensable mass fraction. The submitters involving the remaining five equations per volume is factored into lower and upper submitters, the submitters manipulated to obtain the inverse elements for the row of the matrix defining the pressure. Using the inverse elements, an expression can be obtained for one equation for each volume involving the pressure and the velocities from the attached junctions. The velocity expressions obtained from the momentum equations are used to eliminate the velocities in the pressure equation obtained from the inverse elements. This results in a system of equations, one per volume, involving only volume pressures. The resulting matrix is solved for pressures using a sparse matrix routine. The number of equations in the sparse matrix is one equation for each volume and the nonusers in the sparse matrix are the diagonal element plus an off-diagonal element for each volume. The pressures are back substituted into the expressions from the momentum equations to obtain velocities, and pressures and velocities are back substituted into the other four volume elations to obtain the remaining volume quantities (NSAD, 2001).

The conservative form of the conservation equations should be able to be solved in a similar manner to that for the non-conservative form. However, comparing the non-conservative form, the conservative form of the differential equations introduces five new unknowns, which requires five extra algebraic equations to solve the simultaneous equations. Therefore, there are twenty-one unknowns in total for conservative volume mass and energy sub-matrices. Those unknowns are the convective quantity for non-condensable mass fraction (kg/m^3), the convective quantity for vapor density (kg/m^3), the convective quantity for liquid density (kg/m^3), the convective quantity for vapor internal energy (J/m^3), the convective quantity for liquid internal energy (J/m^3), non-condensable mass fraction, vapor internal energy (J/m^3), liquid internal energy (J/m^3), liquid density (kg/m^3), vapor density (kg/m^3), total pressure (Pa), vapor temperature (K), liquid temperature (K), the saturation temperature corresponding to partial pressure of water vapor (K), the saturation temperature corresponding to total pressure (K), the interface mass transfer (kg/m^{3}s), the interface energy transfer associated with interface mass transfer (W/m^3), the