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Enhance limit cycle oscillation in a wind flow energy harvester system with fractional order derivatives

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Abstract Advances in material science and mathematics in conjunction with technological needs have triggered the use of material and electric components with fractional order physical properties. This paper considers the mathematical model of a piezoelectric wind flow energy harvester system for which the capacitance of the piezoelectric material has fractional order current-voltage characteristics. Additionally the mechanical element is assumed to have fractional order damping. The analysis is focused on the effects of order of derivatives on the appearance and characteristics of limit circle oscillations (LCO). It is obtained that, the order of derivatives to enhance the amplitude of LCO and lower the threshold condition leading to LCO. The domains of efficiency of the system are illustrated in various parameters spaces.

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Scientific research in the area of energy harvester system (EHS) is mainly focused on enhancing the efficiency of the system by developing advanced material and/or considering nonlinear effects to reach widening the frequency bandwidth of the systems.[1–6] Recent advances in material science and nonlinear analysis showed that, material with fractional order properties have deep impacts on systems performance; this includes improving their performance (reduce or enhance mechanical vibration).[7–15] Consequently, potential applications for energy harvesters can be developed. In a recent contribution, we considered material with fractional order deflection and material with fractional order derivative[5] and estimated the best order of deflection and derivative for efficient harvesting. In another contribution, we discussed the effects of fractional order damping on the nonlinear response of a Duffing oscillator showing memory effects.[16]

The main goal of this letter is to discuss the performance of a wind flow energy harvesting system by focusing on the effects of order of fractional derivatives on the existence and amplitude of limit cycle oscillations (LCO). Limit cycle oscillations were reported by Erturk et al.[17] while analyzing a harvester response under uniform airflow. The cut-in wind speed (leading to LCO) was observed experimentally by Kwon[18] for a T-shaped piezoelectric cantilevered beam under axial fluid flow. We discussed the performance of an harvester made of flexible beam with piezo-

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electric patches;[19] we showed that, stochastic resonance helps to overcome the potential barrier for a system made of flexible beam with axial load.[19] Barrero-Gil et al.[20] considered a theoretical model of transverse galloping and determined the relation between the mass and geometric and physical properties of the system and the flow velocity for energy efciency. They concluded that an optimal device should have higher value of the linear coefficient and a low absolute value of the cubic coefficient. Abdelkefi et al.[21] considered galloping with low and high Reynolds numbers. They concluded that the electrical load impedance and the Reynolds number are key parameters in determining the onset of galloping and the harvested power. Evidently, the harvested energy at high Reynolds numbers is much larger than that at low Reynolds numbers.

Continuing the above discussion we focused on wing flow and we examine the specific outcome of integrating into the system components whose characteristics has fractional derivative. Two specific modifications are considered. (1) The capacitance of the piezoelectric patches are supposed to be of the fractional order.[7,8,22] (2) The mechanical part of the system has fractional order damping.[16] The generic physical set up of the system is shown in Fig. 1, where Y is the defection of the magnetoelastic beam of length ℓ, V is the voltage across the load resistor R_ℓ, $F(t)$ is the perturbation due to wind flow, and C_p is the capacitance of the fractance capacitor whose current voltage characteristic is given by $i = C_\mathrm{p}\,\mathrm{d}^\kappa V/\mathrm{d}\tau^\kappa$,[7,8,10,22] where i is the harvested current, τ is the time, and κ is the order of the derivative ($\kappa = 1$ for an ideal piezomaterial). Experiments on PZT show that the polarization falls under compressive stress and this phenomenon generally induces a degradation of the dielectric and piezoelectric properties due to ferroelectric-ferroelastic domain rearrangement.[23]

In the present case the fractional derivative is coming from multiple relaxation spectra of electrical dipoles dynamics. Variable stress load in time and also along the piezoelectric patches, layered structure, properties of the electrodes, and local temperature gradients lead to the hysteretic losses.

We suppose in these analysis that, the fractional derivative is defined according to Caputo definition as[8] $\mathrm{d}^\kappa V/\mathrm{d}\tau^\kappa = \mathrm{D}^\kappa V(\tau) = \int_0^\tau (\tau - s)^{-\kappa}\Gamma^{-1}(1-\kappa)V(s)\,\mathrm{d}s$. $\Gamma()$ is the Gamma function and $n-1 < \kappa < n$, $n \in \{1,2\}$ for all fractional order derivatives in this paper.

The mathematical model is simply given by

$$m\left(Y'' + 2\zeta Y' + \omega_\mathrm{n}^2 Y\right) = \theta V + V_a + F(\tau), \tag{1}$$

$$C_\mathrm{p}\mathrm{D}^\kappa V + V/R = -\theta Y', \tag{2}$$

where the prime denotes the derivation with respect to time, m is the mass of the mechanical element, ζ is the mechanical damping ratio, ω_n is the natural frequency, θ is the electromechanical coupling coefficient, Y is a tip point horizontal displacement, τ is time, ℓ is the structure length, C_p is the capacitance of the piezoelement, R is the load resistance, V is the harvested electric voltage, and $F(\tau)$ is the aerodynamic force, whose expression is given as[19–21,24] $F(\tau) = \rho U^2 D_0\left[a_1 Y'/U + a_3(Y'/U)^3\right]/2$ with ρ being the fluid density, U, the wind velocity, D_0 is the characteristic dimension of the body normal to the incoming flow and a_1 and a_3 are fitting coefficients whose value depends on the angle of attack of the fluid and the Reynolds number.[20,21,24]