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An energy density zone model for fatigue life prediction accounting for non-equilibrium and non-homogeneity effects

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ABSTRACT

An energy density zone (EDZ) model for the description of fatigue crack growth from micro-scale to macro-scale is developed. Five parameters in the model are scale functions that depend on the fatigue crack size or time. Therefore, the non-equilibrium and non-homogeneity (NENH) effects in a fatigue failure process can be reflected. 40CrNi2Si2MoVA steel material is a low alloy and super-high strength steel material. The trans-scale fatigue behaviors are investigated. Five scale functions in the EDZ model are determined from the *S–N* test data for the smooth specimens with the stress concentration factor $K_t = 1$. When the microscopic effects are taken into account, the scattered fatigue test data are precisely re-produced by the proposed model. Then, the fatigue lives for the notched specimens with different values of K_t (i.e., $K_t = 2, 3$) are predicted. In addition, the influences of microscopic effects on the fatigue behaviors are discussed. The results show that the initial micro-defects and the evolution of material micro-structures have a large influence on the material fatigue life. This is the main reason of the scatter of fatigue test data. Otherwise, the NENH effects in a fatigue failure process can be indeed reflected by the EDZ model.

1. Introduction

Any material is both non-homogeneous and non-continuous at the microscopic scale (or micro-scale). Note that the micro-scale in this work specifically indicates the scale of material micro-structures. Fatigue test data has definitely proved that the microscopic effects have a pronounced influence on the fatigue life. In fact, the fatigue failure problem is a multi-scale problem that at least passes two different scales from micro-scale to macro-scale. In addition, the fatigue failure is also a non-equilibrium process [1,2]. Many fatigue analysis approaches have been developed in the past decades based on the classical continuum mechanics. Unfortunately, the classical continuum mechanics based on the hypothesis of homogeneity and continuity cannot take the effects of microscopic inhomogeneity and discontinuity into account. Therefore, fatigue analysis approaches within the framework of classical continuum mechanics cannot consider the non-equilibrium and non-homogeneity (NENH) effects in a fatigue failure process.

In order to consider the scaling hierarchy associated with non-equilibrium in a material failure process, the restraining stress zone was used to connect the defects at different scales such as macro-, micro- and nano-scale. A group of multi-scale crack models has been developed in the last ten years by Sih and Tang [3–12]. The interaction between the defects at different scales was investigated. The basic idea is that a non-equilibrium system can be divided into several equilibrium segments. In this way, the microscopic inhomogeneity can be included into a macro-crack model. These results [3–12] reveal that any material failure process depends on loading, geometry and material property at both micro-scale and macro-scale. Then, the expression of a dual-scale strain energy density factor S_{micro}^{macro} was obtained [13] in which three basic normalized parameters μ^*, σ^* and d^* were involved. Here, μ^* is associated with the microscopic and macroscopic elastic property of a material. σ^* represents the external loading and the material damage. d^* deals with the geometry of macroscopic specimen and material micro-structures. It has been found that the dual-scale energy density factor S_{micro}^{macro} can serve as a controlling quantity of the material failure from micro-scale to macro-scale and has been successfully applied to depict the fatigue multiscaling behaviors [14–21]. Sih [22] concluded in fact that three normalized parameters μ^*, σ^* and d^* are scale transitional functions and depend on the crack size a or time t. Moreover, any material elastic constants should be removed from the new formulation such that the new approach can overcome the limitation of classical continuum mechanics because the classical elastic constants such as the Young's modulus E and Poisson's ratio ν represent the macroscopically homogenized property of a material. Sih also proposed that the term ''restraining stress zone'' used in

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the previous work should be replaced by "energy density zone" (EDZ) in order to avoid any misunderstanding with the analytical approaches based on classical continuum mechanics. It has been demonstrated that the EDZ model can be applied to more common cases including the NENH systems [19–22].

The fatigue behaviors of a low-alloy and super-high strength steel material, 40CrNi2Si2MoVA, are investigated based on the EDZ model in this work. There are five parameters including μ^*, σ^*, d^*, B and m in the EDZ model. Three normalized parameters μ^*, σ^* and d^* are scale transitional functions as discussed early [19–22]. Moreover, it is further pointed out in this work that two other parameters B and m representing the material fatigue property should also be scale transitional functions depending on the location and time.

2. Underlying physics of the EDZ model

The distributed micro-flaws prevail in a material at the micro-scale that cannot be seen by naked eyes. A fractograph showing micro-cavities at the grain boundaries of a steel material is given in Fig. 1 [23]. The size of micro-cavities in the order of magnitudes should be equivalent to the averaged length of micro-structures of a material. The fatigue behaviors of cylinder specimens will be investigated in this work. A cylinder specimen is subjected to the tension–tension fatigue loading $\sigma(t)$ as illustrated in Fig. 2. The fatigue micro-crack nucleates from one or several micro-defects (a microscopic weakened zone). This site is called as the fatigue source as shown in Fig. 2(a). Then, the fatigue micro-crack initiates and propagates as shown in Fig. 2(b). When the fatigue crack can be seen by naked eyes, the fatigue macro-crack initiates and propagates to the final macro-break as shown in Fig. 2(c). Note that the fatigue failure process is the one continuous process that passes two different scale from micro-scale to macro-scale. This scale transitional characteristic is the inherent nature of a fatigue process that can be depicted by the EDZ model.

For any deformed body, the strain energy density field distributes in the body. At the micro-scale, the intensification of the energy density field ahead of a micro-flaw tip is represented by

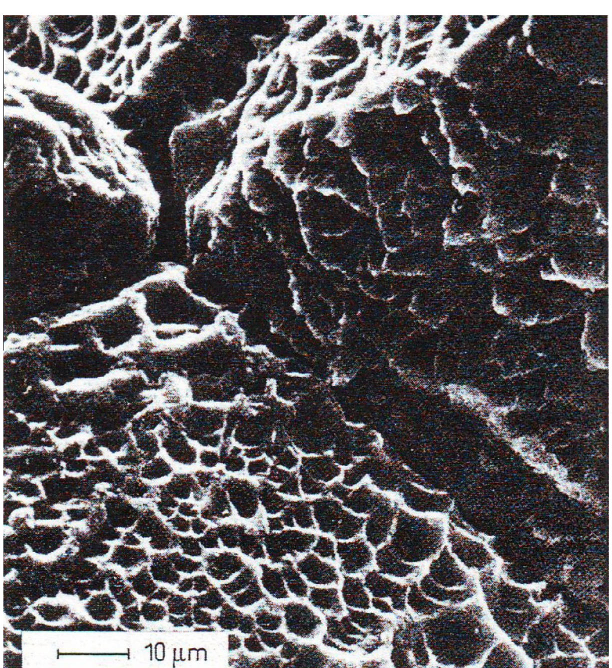

Fig. 1. A fractograph showing micro-cavities at the grain boundaries of a steel [23].

an energy density factor S_{micro}^{macro}. S_{micro}^{macro} can serve as a controlling parameter for the crack growth from micro-scale to macro-scale. The singularity order of energy density field would be weaker or stronger than the normal r^{-1} singularity at the macro-scale depending on loading/restraining, geometry and material property at both micro-scale and macro-scale [5,6]. The material failure is synthetically related to loading/restraining, geometry and material property that can be involved into three normalized parameters in the EDZ model. They are, in fact, the scaling functions. Assuming the micro-scale as a reference scale, three scaling functions can be expressed as [20,22]

$$\mu' = \mu_{ma/mi}, \quad \sigma' = \sigma_{ma/mi}, \quad d' = d_{ma/mi} \tag{1}$$

The macroscopic and microscopic stiffness property is represented by normalized shear modulus μ'. The normalized stress ratio σ' stands for the ratio of applied macro-stress to local restraining stress. The normalized length ratio d' denotes the ratio of a macro-length to a micro-length. Eq. (1) can be regarded as the normalization of macro–micro effects. They describe the transitory behavior of micro–macro cracking.

Assuming the macro-scale as a reference scale, three transitional functions should be then written as

$$\mu'' = \mu_{mi/ma}, \quad \sigma'' = \sigma_{mi/ma}, \quad d'' = d_{mi/ma} \tag{2}$$

Note that Eq. (2) are not equal to Eq. (1) because the reference states are different.

The volume energy density factor (VEDF) is denoted by the symbol S. Here, $S_{ma/mi}$ and $S_{mi/ma}$ apply, respectively, to VEDF of macro → micro and VEDF of micro → macro. They can be regarded as the conversion of ordinary functions S_{ma} and S_{mi} to transitional functions $S_{ma/mi}$ or $S_{mi/ma}$ [24]. A combination of ordinary and transitional functions can be written as

$$k_{ma}S_{ma/mi} = \sigma_{ma}^2 a_{ma}\mu_{ma/mi}(1 - \sigma_{ma/mi})^2 d_{ma/mi}^{1/2} \tag{3}$$

In Eq. (3), k_{ma} is a dimensional compatibility factor so that $S_{ma/mi}$ would have the expected unit when the results for mono-scale are made to conform to those in multi-scale. By the same token, the unit of $S_{mi/ma}$ would be changed accordingly. The reciprocals of the quotients in Eq. (3) can give

$$k_{mi}S_{mi/ma} = \sigma_{mi}^2 a_{mi}\mu_{mi/ma}(1 - \sigma_{mi/ma})^2 d_{mi/ma}^{3/4} \tag{4}$$

where k_{mi} is a microscopic dimensional compatibility factor. To reiterate, micro → macro is not the same as macro → micro. Eq. (3) for the macro-crack corresponds to an intensified macro-stress localization with the distance order of $r^{1/2}$ which tends to be unbounded as the distance r approaches the point of stress localization. Eq. (4) shows that this effect is much more severe for the micro-crack, where the micro-stress localization with the distance order of $r^{3/4}$. According to the scaling segmentation of the stress singularity, the hotspots are connected with the micro–nano stress localization, where the exponent of r is likely to fall between 0.75 and 1.00. For more details, please refer to [24].

In what follows, the multi-scale fatigue behaviors of a steel material will be investigated. The micro-scale (i.e., the scale of material micro-structures) is taken as the reference scale. For clarity, Eq. (3) is re-written as

$$S_{micro}^{macro} = Aa(\sigma_\infty)^2\mu^*(1 - \sigma^*)^2\sqrt{d^*} \tag{5}$$

where

$$\mu^* = \frac{\mu_{micro}}{\mu_{macro}}, \quad \sigma^* = \frac{\sigma_0}{\sigma_\infty}, \quad d^* = \frac{d}{d_0} \tag{6}$$

Here, the symbol S_{micro}^{macro} stands for the scale transitional volume energy density factor from micro-scale to macro-scale in contrast