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Characteristics of crack front stress fields in three-dimensional single edge cracked plate specimens under general loading conditions

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ABSTRACT

In the present article, extensive three-dimensional finite element analyses are conducted for elastic single edge cracked plate (SECP) specimens, which is one of the most widely used specimen for fracture toughness testing. A wide range of geometrical parameters variations are considered including in-plane crack depth to plate width ratio (a/W) and out-of-plane plate thickness to width ratio (t/W). Furthermore, for each three-dimensional specimen, four different crack loading conditions are considered including uniform, linear, parabolic and cubic stress distributions applied on the crack surface. Complete solutions of stress intensity factor, in-plane T-stress (T_{11}) and out-of-plane T-stress (T_{33}) stresses along the edge-crack front are obtained. Characterizations of crack front stress fields using stress intensity factor and T_{11} and T_{33} parameters are carried out. By using superposition method, the present solutions enable the calculations of the stress intensity factors, and T-stresses for wide range of $a/W, t/W$ and under generally loading conditions. Application of this method for the determination of these fracture mechanics parameters for other arbitrary loading conditions, are carried out. The combination of the effects of crack depths (a/W), plate thickness (t/W), and loading conditions on the crack front distributions of stress intensity factor, T_{11} and T_{33} stress are thus discussed. Solutions obtained will be very useful for analyzing fracture toughness test data for single edge cracked plate specimens with different crack depths (a/W), thickness ratios (t/W), and under general loading conditions.

1. Introduction

It is well known that the fracture resistance (toughness), which is the critical value of the crack driving force (K or J), is dependent on the constraint at the crack tip for a given cracked specimen. The conventional one-parameter fracture analysis is based on the fact that the crack has to be highly constrained so that the minimum value of the material toughness can be achieved. This will ensure the fracture assessment is always on the safe side. But it may cause overly conservative assessment for the structures that are not highly-constrained, and thus raises the cost of product and maintenance.

In a three-dimensional isotopic linear elastic body containing a crack subjected to symmetric (mode I) loading, as shown in Fig. 1, the Williams series expansion for linear elastic crack-tip fields, including only the singular term and constant term, can be written as [1–3]:

$$\sigma_{11}(r,\theta) = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+T_{11} \tag{1a}$$

$$\sigma_{22}(r,\theta) = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{1b}$$

$$\sigma_{33}(r,\theta) = \frac{K}{\sqrt{2\pi r}}2\nu\cos\frac{\theta}{2}+T_{33} \tag{1c}$$

$$\sigma_{12}(r,\theta) = \frac{K}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{1d}$$

$$\sigma_{13}(r,\theta) = T_{13} \tag{1e}$$

$$\sigma_{23}(r,\theta) = 0 \tag{1f}$$

where r and θ are in-plane polar coordinates of plane normal to the crack front and the subscripts 1, 2 and 3 suggests a local Cartesian co-ordinate system formed by the plane normal to the crack front and the plane tangential to the crack front point. ν is the Poisson's ratio. The magnitude of the singular term K is the local mode I stress intensity factor. There are three non-zero components involved in

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the constant T-stress term: T_{11}, T_{33} and T_{13}. In practice, T_{11} is usually simply called the T-stress, which represents a tensile or compressive stress acting in the crack plane and perpendicular to the crack front; while T_{33} represents the stress acting in the crack plane, and tangent to the crack front. The third T-stress components T_{13} is the shear stress perpendicular to the crack plane. These three components are independent of each other for three-dimensional crack fronts. It can be shown further that T_{33} and T_{11} are related through the following relationship [2,3]:

$$T_{33} = E\varepsilon_{33} + \nu T_{11} \quad (2)$$

where ε_{33} is the tangential strain component at the crack front. Note under two-dimensional plane strain or plane stress conditions, there is only one independent T-stress component (i.e. T_{11}). For plane strain, $T_{33} = \nu T_{11}$ and $T_{13} = 0$; while for plane stress, $T_{33} = T_{13} = 0$. Only T_{11} and T_{33} will be discussed in present work.

Using two-dimensional plane strain models, Larsson and Carlsson [4] and Rice [5] showed that the magnitude of the T-stress affects the size and shape of the plastic zone and the region of tensile triaxiality ahead of the crack tip. If the T_{11} stress is positive, J-dominance is maintained and a single parameter J can be used for a fracture criterion. If the T_{11} stress is negative, a two-parameter approach (J and T_{11}) is required to characterize the stress fields. Bilby et al. [6], Betegon and Hancock [7], Du and Hancock [8], O'Dowd and Shih [9] demonstrated that the T_{11}-stress and J-integral provide effective two-parameter characterization of plane strain elastic–plastic crack tip fields in a variety of crack configuration and loading conditions.

For three-dimensional crack problems, Rice [5] suggested that both T_{11} and T_{33} stresses should be considered as the constraint parameters in order to characterize the stress field in the vicinity of crack front. Subsequently, Wang [10] and Gonzalez-Albuixech et al. [11] showed that the consideration of the components T_{11} and T_{33} was necessary to obtain a correct description of the stress state near the crack front. Recently, there have been thriving interests in studying quantification of stress fields for three dimensional crack front including the effects from both components of T-stresses, see [12], for example.

To quantify the effects of these parameters on fracture properties, further computational study and experimental testing is required. The single edge cracked plate (SECP) specimen is a common test specimen in research and practice of fracture mechanics [13], see Fig. 2. Three point bend loading is the most commonly used loading conditions, with the thickness to width ratio t/W of 1. Furthermore, in recently years, tension loading has also been employed to obtain the test toughness in low constraint conditions, see Cravero and Ruggieri [14] and Shen and Tyson [15], for example. On the other hand, the varying thickness-to-width ratios t/W is also investigated to study the thickness effect on fracture toughness [16].

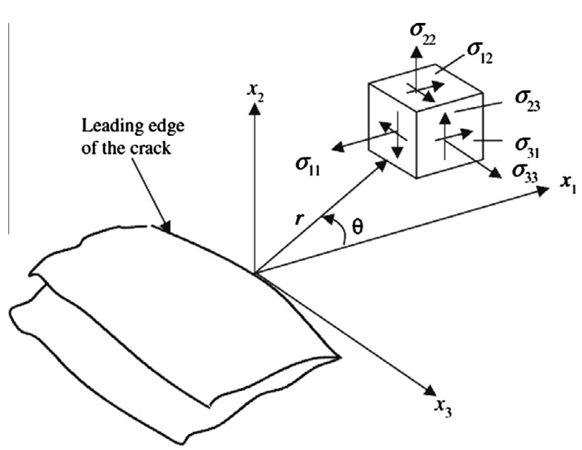

Fig. 1. Three-dimensional coordinate system for the region along the crack front.

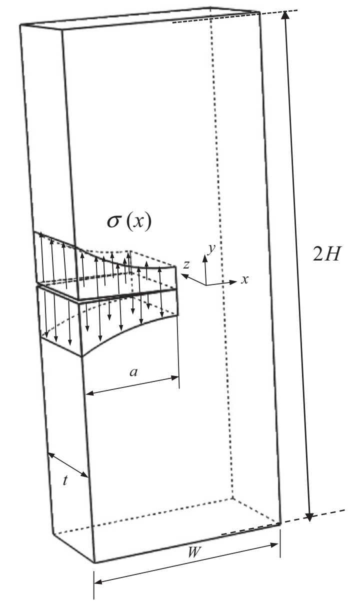

Fig. 2. 3D SECP specimen.

Most previous stress intensity factor and T-stress solutions for SECP specimens are for two-dimensional plane-strain or plane-stress cases [13,17–20]. However, the real test specimens are three-dimensional, and the stress intensity factors and T-stresses vary along the crack front. Furthermore, the solutions depend on geometrical parameters including in-plane dimensions, a/W (shallow or deep cracks), and out-of-plane dimensions, t/W (thin specimen or thick specimen), and the loading conditions (such as tension or bending). Complete three-dimensional analyses are required to study the all the effects together. Over the last twenty years, several three-dimensional finite element analyses of the 3D through-the-thickness test specimens were carried out in the literature. For example, Kwon and Sun [21] studied the 3D effects on stress intensity factors for center cracked tension specimens. Kacianauskas et al. [22] calculated the stress intensity factors for single edge cracked specimens under bending loads by three-dimensional finite element analyses. Three dimensional stress intensity factors are also obtained for remote tension loadings for $t/W = 1$ [15]. Solutions for stress states and stress intensity factors were obtained for very thin plates [23]. Most recently, Lu and Meshii [24] have calculated the T-stresses for bending loads with varying t/W ratios. Due to the complexity of the 3D problem, each of these studies has focused on limited geometry or loading conditions and for either stress intensity factors or T-stresses.

In this work, the objectives are to further study computationally the characteristics of the three dimensional crack front stress fields in 3D elastic SECP specimens. We aim to develop the *complete* solutions of stress intensity factors (K) and T-stress components (T_{11} and T_{33}) covering the entire range of a/W and t/W ratios (shallow and deep cracks in thin and thick plates), under arbitrary linear and nonlinear stress distributions. In addition, the characterization of stress fields using three fracture mechanics parameters will be carried out. By using superposition method, the solutions obtained from the present analyses will enable the calculations of the stress intensity factors, and T-stresses for wide range of $a/W, t/W$ and under generally loading conditions. Application of this method for the determination of these fracture mechanics parameters for other stress distributions will be demonstrated. The combined effects of crack depth ratio (a/W), specimen thickness ratio (t/W), and loading conditions will be discussed.