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Fracture analysis of an orthotropic strip with imperfect piezoelectric coating containing multiple defects

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ABSTRACT

This paper presents an analytical model for the analysis of an orthotropic strip with piezoelectric coating weakened by multiple defects. The medium is subjected to anti-plane mechanical and in-plane electrical loading. Using the dislocation method, the problem can be reduced to a system of singular integral equations, in which the unknown variables are dislocation densities. The integral equations are of the Cauchy singular kind and are solved numerically to determine stress intensity factors at the crack tips and hoop stress on the cavity. The obtained results are analyzed to evaluate the dependence of the stress intensity factors and hoop stress on the imperfect bonding coefficient, the defect geometry and material properties of the orthotropic substrate.

1. Introduction

It is well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. Due to this intrinsic coupling phenomenon, piezoelectric materials are used widely in technology such as high-power sonar transducers, electromechanical actuators, and piezoelectric power supplies. When subjected to mechanical and electrical stresses in service, these piezoelectric materials, which are mostly brittle ceramics, can fail prematurely due to the propagation of flaws or defects produced during the manufacturing process. Therefore, the study of the fracture mechanics of piezoelectric materials plays an important role in the design of piezoelectric devices.

To gain advanced performance, piezoelectric components are often made as layered structures. The simplest structure is just composed of a piezoelectric layer and a substrate. Orthotropic composites are sometimes used as the substrate of layered piezoelectric devices to enhance mechanical performance. In the piezoelectric composite, the interface is an important part that is responsible for the transmission of electro-elastic fields between the layer and substrate. This has prompted the undertaking of the present study.

Due to the anisotropic nature of piezoelectric materials and the coupling of an additional field variable in the governing equations, piezoelectric boundary value problems generally pose serious mathematical difficulty. When a defect such as a crack is introduced, the problem becomes even more difficult.

The investigation of interfacial cracks between the embedded electrode layer and piezoelectric ceramic has been performed by Ru [1]. Kwon and Lee [2] studied the problem of an interface crack between piezoelectric and elastic strips. The problem of a central crack normal to a piezoelectric–orthotropic interface has been solved in the paper by Kwon and Meguid [3]. Chi and Chung [4] obtained the stress intensity factor for cracked multi-layered and functionally graded material coatings of a coating-substrate composite. Lee et al. [5] reported the result of an interfacial crack moving along the interface between a piezoelectric and two orthotropic materials. In another reported research [6] anti-plane problem of periodic cracks in a functionally graded coating-substrate structure was studied. The problem of a penny-shaped crack in a piezoelectric fiber with an elastic coating was analyzed by Qin et al. [7]. They studied the effect of the thickness and the elastic material properties of the coating on the fracture behaviour of piezoelectric fiber composites. Jin and Fang [8] treated the problem of the electroelastic interaction between multiple screw dislocations and a circular inclusion with an imperfect interface under anti-plane shear and in-plane electrical field loadings in a piezoelectric material. The problem of FGPM coating containing a permeable crack under anti-plane loading with a Kelvin-type viscoelastic interface, was analyzed by Cheng and Zhao [9]. The effect of an imperfect interface on the fracture behaviour of a layered piezoelectric was studied and reported by Li and Lee [10]. They employed the Fourier integral transform and Cauchy singular integral equation and found that the crack tip shielding and

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anti-shielding effects of the mechanical imperfection are more remarkable than those of the electric imperfection. In another paper Li and Lee [11] studied the concept of weak discontinuity to the interface in FGPMs, and investigated the fracture behavior of a weak discontinuous interface between two piezoelectric strips under electromechanical loads by the methods of Fourier transform. The effects of the viscoelastic interface and FGPM graded properties on stress intensity were studied. Peng and Li [12] analyzed the fracture behaviour of homogeneous magneto electro elastic substrate with a functionally graded coating containing a crack at the interface. Li and Ding [13,14] considered the problem of mode III crack in a functionally graded piezoelectric layer bonded to a piezoelectric half-plane. Feng et al. [15] investigated the problem of multiple cracks on the interface between a piezoelectric layer and orthotropic substrate and showed the stress intensity factor is dependent on the geometrical parameters and material orthotropy. Asadi et al. [16] solved the anti-plane shear problem of orthotropic strips with multiple defects and imperfect FGM coating. They studied effects of material properties of the FGM layer and the spring constant of imperfect boundary on stress fields. Ding and Li [17] investigated the problem of two collinear cracks perpendicular to the interface of functionally graded orthotropic strip bonded to an orthotropic substrate. They studied the effects of the orthotropy and nonhomogeneous parameters on the stress intensity factors. The distributed dislocation technique was also applied to the analysis of cracked functionally graded piezoelectric layer [18] and the interaction of multiple cracks was studied. Bagheri et al. [19] investigated fracture behaviour of an orthotropic strip with an orthotropic functionally graded coating with multiple cracks under anti-plane loading. They obtained stress intensity factors during the passage of a time-harmonic shear wave.

The objective of this paper is to seek the solution of multiple defects problem with arbitrary patterns in an orthotropic strip reinforced with a piezoelectric coating. The bonding between piezoelectric coating and the orthotropic substrate is modeled as a linear spring. The technique necessities the solution of screw Volterra dislocation in the region. The Fourier transform is employed to obtain dislocation solution to formulate integral equations for a coating-substrate structures weakened by multiple defects. The integral equations are of Cauchy singular type. The resulting singular integral equations were solved using the collocation method, developed by Erdogan and Gupta [20], to provide the dislocation density on the cracks faces. Numerical results are presented to show the influences of the orthotropy parameter, bonding coefficient and defect geometry on the resulting stress intensity factors and hoop stress, which may provide references for the design and assessment of this kind of smart structures.

2. Problem formulation

Table 1
Material properties for numerical examples.

	PZT-4	PZT-5H
$c_{44}\left(\frac{N}{m^2}\right)$	2.56×10^{10}	3.53×10^{10}
$e_{15}\left(\frac{C}{m^2}\right)$	12.7	17
$d_{11}\left(\frac{C}{Vm}\right)$	64.6×10^{-10}	151×10^{-10}

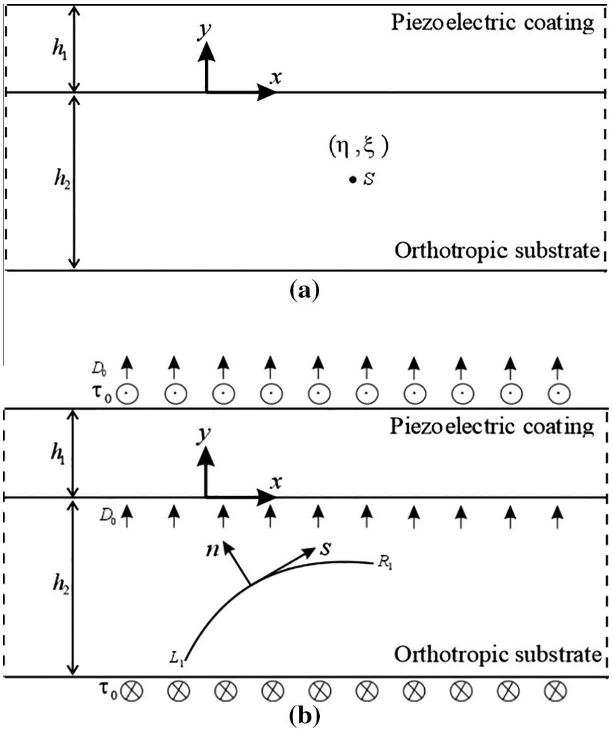

Fig. 1. Schematic view of an orthotropic layer with piezoelectric coating. (a) Screw dislocation. (b) Curved crack.

A Volterra type screw dislocation with Burgers vector b_{wz} be situated at a point (η, ξ) in the substrate with the piezoelectric coating (Fig. 1a). The x-and y-axis are in the directions of principal material orthotropy of orthotropic substrate. The electroelastic boundary value problem is simplified considerably if we consider only the out of plane displacement and in-plane electric fields. Since piezoelectric layer is assumed to be polarized in z-axis, the problems involves the anti-plane elastic field coupled with the in-plane electric field. Therefore, the constitutive relations of the problem in coordinate system (x, y, z) can be written as follows:

$$\sigma_{xz} = c_{44}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \phi}{\partial x}$$

$$\sigma_{yz} = c_{44}\frac{\partial w}{\partial y} + e_{15}\frac{\partial \phi}{\partial y}$$

$$D_x = e_{15}\frac{\partial w}{\partial x} - d_{11}\frac{\partial \phi}{\partial x}$$

$$D_y = e_{15}\frac{\partial w}{\partial y} - d_{11}\frac{\partial \phi}{\partial y} \quad 0 \leqslant y \leqslant h_1 \tag{1}$$

where D_x, D_y and ϕ are the electric displacements, and electric potential, respectively. Also c_{44} is the elastic stiffness measured in a constant electric field, e_{15} is the piezoelectric constant and d_{11} is the dielectric measured at a constant strain. The constitutive relations may be written as

$$\sigma_{zx}(x, y) = G_x\frac{\partial w}{\partial x}, \quad \sigma_{zy}(x, y) = G_y\frac{\partial w}{\partial y} \quad -h_2 \leqslant y \leqslant 0 \tag{2}$$

In above equalities, G_x and G_y are the orthotropic shear moduli of elasticity of material. Combining Eqs. (1) and (2), in the absence of body forces and free charges, the equilibrium equations and Maxwell's equation for the piezoelectric layer and orthotropic substrate under anti-plane mechanical and in-plane electrical conditions, reduce to

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0, \quad e_{15}\nabla^2 w - d_{11}\nabla^2\phi = 0 \quad 0 \leqslant y \leqslant h_1$$

$$\frac{\partial^2 w}{\partial y^2} + g^2\frac{\partial^2 w}{\partial x^2} = 0 \quad -h_2 \leqslant y \leqslant 0 \tag{3}$$