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Effect of electrostatic tractions on the fracture behavior of a piezoelectric material under mechanical and/or electric loading

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ABSTRACT

The pre-cracked parallel-plate capacitor model is further developed to study analytically the effect of electrostatic tractions induced by Maxwell stress and piezoelectricity on the fracture behavior of a piezoelectric material under mechanical and/or electric loading. The results indicate that the Maxwell stress and piezoelectricity induced tractions are independent and dependent on the direction of an applied electric field, respectively. Hysteresis loops in the curves of crack opening (or closing) versus applied mechanical strain and in the curves of crack opening (or closing) versus applied electric field occur under positive fields much easier than negative fields due to the piezoelectricity. Because of the potential presence of hysteresis loops, the fracture criterion must be composed of two parts: the energy release rate must exceed a critical value and the mechanical load must be higher than the critical value for crack opening.

1. Introduction

The widely used piezoelectric materials are ferroelectric ceramics, which have become widely adopted in various applications, such as in memory devices, sensors, actuators, transducers and energy harvesting, due to their extraordinary dielectric and piezoelectric properties. Piezoelectric ceramics are brittle and fracture is one of the major failure modes. The fracture behavior of piezoelectric materials under mechanical and/or electrical loading has been studied intensively, see review articles [1–3] and references wherein for details. However, the traction-free boundary condition is usually used in the study of piezoelectric fracture [4–6]. For an electrically insulating crack, electric field exists inside the crack and thus makes the J-integral, J, differ from the energy release rate, G [6–8]. Under combined mechanical and/or electric loading, induced charges merge along the faces of an electrically insulating crack due to the difference in dielectric constants between piezoelectric material and air (or vacuum) inside the crack. Gao et al. [9] theoretically investigated electrostatic force between the induced charges along crack faces and its influence on the piezoelectric fracture behavior. To force $G = J$ and use the opened crack profile, Landis [10] proposed the energetically consistent approach, which gave that the electrostatic traction should be used as the mechanical boundary condition along crack faces in piezoelectric materials. After that, many researchers [11,12] re-analyzed the fracture problem of an electric insulating crack in an infinitely large piezoelectric material and examined the role of electrostatic tractions. Fan et al. [13] discussed in detail the energetically consistent approach and electrostatic tractions along crack faces and along infinite boundaries, which are surrounded by air or vacuum. Their discussions indicate that there might be two solutions for crack opening displacement during a region of the combined applied stress and electric displacement in the energetically consistent approach. Fan et al. [13] defined the threshold applied stress as the minimum applied stress for a given applied electric displacement to open the crack, below which the crack will be closed. The conjugate applied electric displacement of the threshold stress might be called the maximum tolerable field, above which the crack will also be closed. The threshold stress and the maximum tolerable electric displacement determine one end of the region with two solutions of crack opening displacement. The applied stress and its conjugate electric displacement at the other end of the two-crack opening displacement region are called the bifurcation stress and the bifurcation electric displacement, respectively. The electrostatic traction along the infinite boundaries of air or vacuum is equivalent to an additional mechanical load for a given electric load, which is tensile because the dielectric constant of the infinite medium is smaller than that of the material, thereby enlarging crack opening and aiding energy release rate. This might not be realistic, because in experiments and practical electronic applications, electrodes are usually bonded to piezoelectric materials. Electrodes are electric conductors with infinitely large dielectric constants. To analytically investigate the effect of

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electrostatic tractions on the fracture behavior, Zhang and Xie [14] proposed a simple parallel-plate capacitor model, where a semi-infinite long crack lies in the middle of an infinitely long beam. This model takes the advantage in the analysis of energy release rate for steady crack propagation, in which the energy release rate calculated is completely independent of the crack tip location and determined only by the difference between the electric enthalpy far ahead the crack tip and the electric enthalpy far behind the crack tip. Similar parallel-plate capacitor models have been used before by Zhang [15,16] and McMeeking [17] to evaluate the electric contribution to the energy release rate. In the previous pre-cracked parallel-plate capacitor model [14], the capacitor is made of an isotropic dielectric material with rigid electrodes coherently bonded on the capacitor surfaces and the crack interior is air or vacuum. The mechanical and electric loading is implemented by applying mechanical displacement and electric voltage. This simple and ideal model allows one to study analytically electrostatic tractions, deformation, and energy release rate. The analysis indicates that electrostatic tractions on the electrodes compress the material in front of the crack tip, while electrostatic tractions on the crack faces have the tendency to close the crack and stretch the material behind the crack tip. As a result, there exists a threshold of applied mechanical strain to maintain the opening of a crack and there is also an applied bifurcation strain to open a struck crack. Due to the electrostatic traction induced crack sticking, both curves of crack opening displacement versus applied strain and crack opening displacement versus applied field exhibit hysteresis loops. Furthermore, an analytic formula of the failure criterion has been derived based on the energy release rate. The results show that an applied electric field impedes crack propagation, whereas applied mechanical load tends to propagate the crack. The applied electric field will play a more significant role in the fracture behavior, if the material dielectric constant is higher and/or the mechanical fracture toughness of the material in terms of the critical energy release rate is lower. Because of the crack sticking, a crack opening criterion must be added besides the critical energy release rate for crack propagation, meaning that the applied mechanical load must be high enough to open the crack first and then to propagate it. Obviously, the piezoelectric effect is not considered in the previous work [14], the influence of an electric field on the deformation and fracture behavior is through the electrostatic tractions. In the present work, we consider the piezoelectric effect and evaluate the contribution of electrostatic tractions to piezoelectric fracture.

2. Theoretical analysis

2.1. Electrostatic traction induced deformation

Fig. 1 shows the adopted pre-cracked parallel-plate capacitor model. A semi-infinitely long crack is located along the *x*-axis with the origin of a coordinator system at the crack tip. The parallel-plate capacitor is infinitely large in the (x,z) plane, but has an

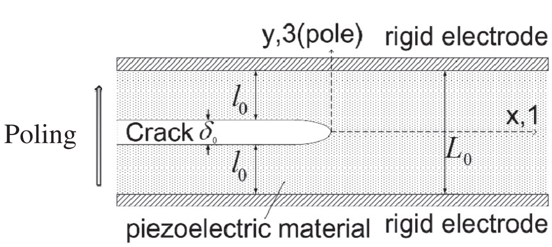

Fig. 1a. Schematic plot of a pre-cracked parallel-plate capacitor with original configuration.

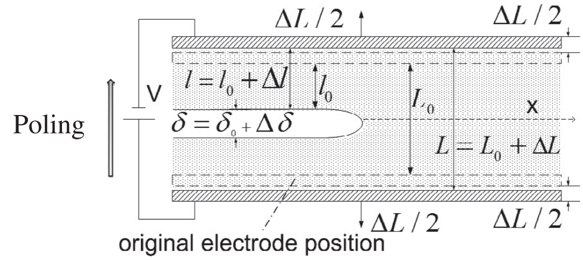

Fig. 1b. Schematic plot of a pre-cracked parallel-plate capacitor under combined mechanical and electric loading.

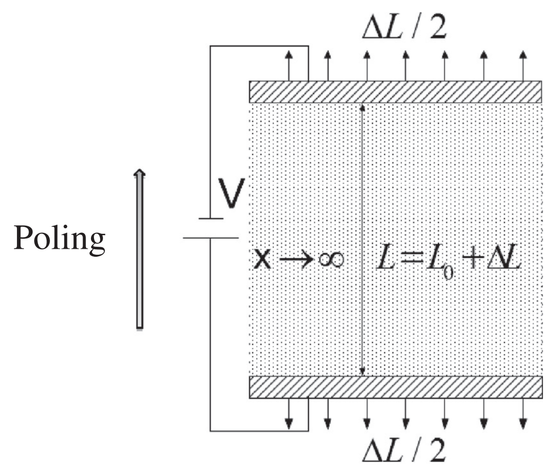

Fig. 1c. Schematic plot of a pre-cracked parallel-plate capacitor far ahead the crack tip.

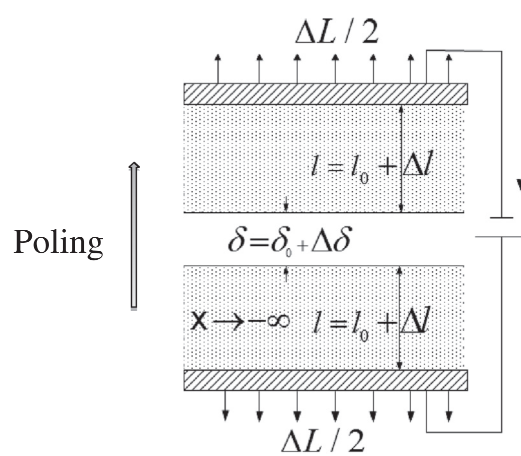

Fig. 1d. Schematic plot of a pre-cracked parallel-plate capacitor far behind the crack tip.

original thickness, L_0, in the uncracked part along the y direction. In the cracked part, each side of the material has an original thickness, l_0, and the crack has an original width, δ_0. The two electrodes are assumed to be mechanically rigid. The poling direction is along the y direction. An electric voltage V and a mechanical displacement ΔL are applied upon the two electrodes. An applied field is called positive or negative when it is parallel or antiparallel to the poling direction. When choosing mechanical displacement ΔL and electric voltage V as independent variables, electric enthalpy, H, will be the appropriate thermodynamics function. In differential form, we have