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# Singular stresses due to adhesion defect on intersection line along which a semi-infinite thin plate is attached to an infinite thin plate by eigenfunction expansion method

D.H. Chen [a], K. Ushijima [b,*]

[a] Department of Mechanical Engineering, Tokyo University of Science, 6-3-1 Niijuku, Katsushika-ku, Tokyo 1258585, Japan
[b] Department of Mechanical Engineering, Kyushu Sangyo University, 2-3-1 Matsukadai, Higashi-ku, Fukuoka 8138503, Japan

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ABSTRACT

In this paper, the stress singularity due to adhesion defect on intersection line along which a semi-infinite thin plate (Plate 1) is attached to an infinite thin plate (Plate 2) is studied by the eigenfunction expansion method. For Plate 1 the stress is approximated as plane stress state, and for Plate 2 the stress is treated as a two-dimensional problem, in which the anti-plane deformation is also taken into account besides the plane stress. The eigenequation for the asymptotic behavior of stresses around the defect tip is given in an explicit form. This eigenequation is different from the analysis where the anti-plane deformation of Plate 2 is ignored. Specifically, it is found that the eigenvalue in consideration of the anti-plane deformation becomes a complex value with a real part equal to 0.5. Also, the singular stress around the defect tip is given in an explicit form. The obtained results are verified through comparison with numerical results of the finite element method.



## 1. Introduction

Over the decades, the sandwich structures like honeycombs which consist of multiple thin-walled plates have been widely used in many structural applications owing to their superior mechanical performance per unit volume. Because of this trend, the precise evaluation of the strength of the sandwich structures has regarded as important, and many structural analyses of sandwich structures have been reported by many researchers [1].

In general, the honeycomb sandwich panels are composed of a core of hollow hexagonal elements with facesheets bonded to each side. Since the performance of the structures essentially depends on the integrity of the adhesive bond between the facesheets and core panels, many studies have been performed on characteristics of bonded intersection, such as stress singularity and interface fractures [3–8].

The stress singularity problems for structures have been investigated by many researchers using theoretical, experimental and numerical approaches. One of the most famous theoretical approaches for stress singularity has been presented by Williams [2]. He uses the eigenfunction expansion method to study the single-material wedge under different boundary conditions. The eigenfunction expansion method is especially suitable for representing the elastostatic singularity in a corner region, and has strong theoretical support. In particular, Gregory [9] has proven that the Williams' eigenfunctions are complete for the annular sector, an issue of both computational and analytical importance. On the other hand, Ural et al. [10] performed experimental study to evaluate the adhesive bond in honeycomb sandwich panels. They investigated the tensile strength and fracture toughness of the facesheet/core bond that needs for the numerical works defining the delamination behavior of panels. Also, Hohe and Becker [11], Hohe et al. [12,13] concerned with the singularity of the stress fields in the cell walls of hexagonal honeycomb cores using a closed form asymptotic analysis and the finite element method. They found the stress singularity at the intersection to be of the pure power-law type. The bonded edge between the facesheet and the core panel for a honeycomb can be recognized as a T-shaped junction which consists of two thin-walled plates. Also, the T-shaped junction can be observed everywhere in a conventional honeycomb structure.

On the contrary, in manufacturing process of a honeycomb sandwich panel, adhesion defects may occur where the core does not successfully bond to the facesheets. However, little studies on an adhesion defect problem have been reported up to now. One of our authors has reported the general solution of stresses due to an adhesion defect for a T-shaped junction, in which a semi-infinite thin plate (Plate 1) is attached to an infinite thin plate (Plate 2), under the supposition that the stresses in each plate can be approximated as the plane stress condition [14].

* Corresponding author. Tel.: +81 926735605; fax: +81 926735090.
*E-mail addresses:* chend@rs.kagu.tus.ac.jp (D.H. Chen), kuniharu@ip.kyusan-u.ac.jp (K. Ushijima).

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However, as an actual applied problem, the necessity of taking the anti-plane deformation of Plate 2 into consideration comes out frequently. Therefore, the singular stress field near the vertex of an adhesion defect in a T-shaped junction of two plates as shown in Fig. 1 is investigated with taking the anti-plane deformation into consideration. In Fig. 1, the Plate 1 can be regarded as a wedge with its angle $\gamma = 180^\circ$, and is bonded perpendicularly to the Plate 2 along the edge OA. The edge OB corresponds to an adhesion defect such as a crack of Plate 2 and no stresses can be observed along the edge. Here, the thickness, shear modulus and Poisson's ratio for each plate are described by parameters $h, G$ and $\nu$, respectively. Also, in the remainder of this paper, quantities with subscripts 1 and 2 denote the properties of the respective plates, while quantities without subscripts denote the common characteristics of the adhered plates.

The adhesion defect problem addressed in this study may be regarded as one of the conventional interface crack (or defect) problems investigated by many researchers [15][16]). However, the adhesion defect discussed in this paper lies along the interface between two thin plates, where each plate stands in a different coordinate plane. Therefore, the conventional solutions for the interface defect problems cannot be applied to this problem.

Essentially, the problem shown in Fig. 1 can be regarded as a three-dimensional stress singularity problem. However, if the thicknesses $h_1$ and $h_2$ is sufficiently small in comparison with the analyzed region, such as a process zone for dominating the fracture toughness, the characteristics of stress distribution in the region can be evaluated from the two-dimensional singular stress fields near the vertex of the point O for these two plates. Moreover, when the in-plane load is applied to the Plate 1, the stress state for the Plate 1 can be approximated under plane stress condition. However, as for the Plate 2, this is a two-dimensional problem considering the anti-plane deformation. Based on this assumption, in this paper, the theoretical analysis for the stress singularity around the defect tip of T-shaped junction is conducted using the eigenfunction expansion method, and the theoretical results are compared with the numerical results obtained by FE analysis. In order to investigate this problem, the development of a new analytical technique is needed, which is the main objective of this study.

## 2. Method of analysis

### 2.1. Expressing stress equations using complex stress functions

According to plate deformation theory with consideration of conventional shear deformation, the displacement of a plate (in-plane displacement $u(x,y,t)$ and $v(x,y,t)$ in the $x$ and $y$ directions, and the displacement in the thickness direction $w(x,y,t)$ ) can be assumed as follows:

$$\begin{cases} u(x,y,t) = u_0(x,y) + t\bar{u}(x,y) \\ v(x,y,t) = v_0(x,y) + t\bar{v}(x,y) \\ w(x,y,t) = w_0(x,y) \end{cases} \tag{1}$$

Here, $t$ is the coordinate in the thickness direction from the middle surface of the plate.

The strains can be given by Eq. (1) as:

$$\begin{cases} \varepsilon_x = u_{0,x}(x,y) + t\bar{u}_{,x}(x,y) \\ \varepsilon_y = v_{0,y}(x,y) + t\bar{v}_{,y}(x,y) \\ \gamma_{xy} = \left(u_{0,y}(x,y) + v_{0,x}(x,y)\right) + t\left(\bar{u}_{,y}(x,y) + \bar{v}_{,x}(x,y)\right) \end{cases} \tag{2}$$

and

$$\begin{cases} \gamma_{zx} = \bar{u}(x,y) + w_{0,x}(x,y) \\ \gamma_{zy} = \bar{v}(x,y) + w_{0,y}(x,y) \end{cases} \tag{3}$$

Therefore, based on the principle of virtual work, equilibrium equations for in-plane stresses at the middle surface of a plate $(\sigma_{x0}, \sigma_{y0}, \tau_{xy0})$ and anti-plane shear stresses $(\tau_{zx}, \tau_{zy})$ can be derived as follows:

$$\begin{cases} \frac{\partial \sigma_{x0}}{\partial x} + \frac{\partial \tau_{xy0}}{\partial y} = 0 \\ \frac{\partial \sigma_{y0}}{\partial y} + \frac{\partial \tau_{xy0}}{\partial x} = 0 \end{cases} \tag{4}$$

and

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0 \tag{5}$$

The strains related to the in-plane stresses $\sigma_{x0},\ \sigma_{y0},\ \tau_{xy0}$ at the middle surface consist only of the in-plane strain in the middle surface, and are related to only the terms $u_0(x,y)$ and $v_0(x,y)$ in Eq. (1) as:

$$\begin{cases} \varepsilon_{x0} = u_{0,x}(x,y) \\ \varepsilon_{y0} = v_{0,y}(x,y) \\ \gamma_{xy0} = u_{0,y}(x,y) + v_{0,x}(x,y) \end{cases} \tag{6}$$

As Eqs. (4) and (6) suggest, even through the anti-plane shear stress of the plate is considered, the in-plane stress at the middle surface of the plate $\sigma_{x0},\ \sigma_{y0},\ \tau_{xy0}$ and the in-plane displacement $u_0,\ v_0$ are expressed in terms of two complex stress functions $\phi(z)$ and $\psi(z)$ as in the case of a two-dimensional elastic problem. Thus, the following equations must be satisfied.

$$\begin{cases} \sigma_x + \sigma_y = 4Re[\phi'(z)] \\ \sigma_y - \sigma_x + 2i\tau_{xy} = 2\{\bar{z}\phi''(z) + \psi'(z)\} \end{cases} \tag{7}$$

for in-plane stress components, and

$$2(u + iv) = \frac{1}{G}\left\{\kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)}\right\} \tag{8}$$

for displacements $u$ and $v$. Here, parameter $z$ and $\kappa$ in the above equations can be represented by $z = x + iy$ and $\kappa = (3-\nu)(1+\nu)$, respectively.

In discussing the in-plane stresses and displacements, only those in the middle surface should be considered. Thus, the subscript 0 is omitted in Eqs. (7) and (8).

As for the anti-plane shear stresses $\tau_{zx}$ and $\tau_{zy}$, the corresponding strains $\gamma_{zx}$ and $\gamma_{zy}$ are given by Eq. (3). Here, in order to examine the stress singularity, the displacement functions shown in Eq. (1) that satisfy only the following equations are needed for consideration:

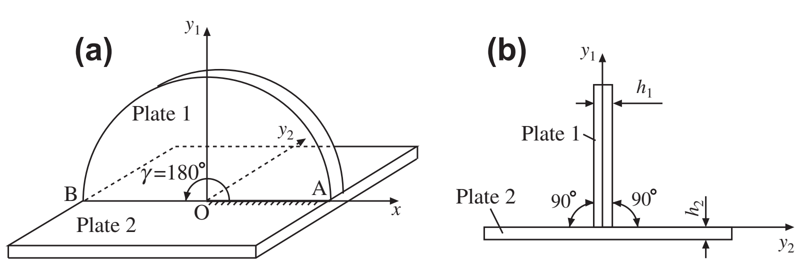


**Fig. 1.** T-shaped junction of two plates.