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Phase-field modeling of fracture in linear thin shells

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ABSTRACT

We present a phase-field model for fracture in Kirchoff–Love thin shells using the local maximum-entropy (LME) meshfree method. Since the crack is a natural outcome of the analysis it does not require an explicit representation and tracking, which is advantage over techniques as the extended finite element method that requires tracking of the crack paths. The geometric description of the shell is based on statistical learning techniques that allow dealing with general point set surfaces avoiding a global parametrization, which can be applied to tackle surfaces of complex geometry and topology. We show the flexibility and robustness of the present methodology for two examples: plate in tension and a set of open connected pipes.

1. Introduction

The prediction of fracture in thin structures is of major importance in engineering applications such as aircraft fuselages, pressure vessels, automobile components, and castings. Since analytical solutions provide limited information, there has been a keen interest in numerically simulating fracture in thin shells in recent years. However, despite the advances made in modeling fracture for solid bodies [1–5], fracture in thin bodies remains a challenge due to the complex interplay between cracks and the shell kinematics and geometry.

Non-propagating cracks in plates and shells have been modeled with partition-of-unity methods [6–8]. These approaches have been restricted to simple geometries. The majority of formulations are based on Mindlin–Reissner theory [9]. There are comparatively fewer methods considering fracture in thin shells [10]. In [11], a shell element based on discrete Kirchhoff theory was proposed assuming through-the-thickness cracks. Later, a shell model with the phantom node method based on edge rotations was proposed [12] for both thin and thick shells, where the crack tip can be located inside an element. A method based on subdivision shell elements and modeling the fracture along the element edges with a cohesive law was proposed in [13]. In [14–16], a meshfree thin shell model for static and dynamic fracture was presented.

Most of above methods are based on discrete crack models that require explicitly (or sometimes implicitly [17]) tracking the crack path. Furthermore, many of the approaches are applied to simple geometries such as plates, or spherical and cylindrical geometries [18,19,12,20]. Towards a general, flexible and robust methodology to deal with fracture in Kirchhoff–Love shells, we propose here treating fracture with a phase-field model and discretizing the coupled thin-shell/phase-field equations with a recently proposed meshfree method for partial differential equations on manifolds of complex geometry and topology [21,22].

Phase-field methods are widely used in science and engineering to model a variety of physics [23–26]. The extension of this method for fracture in solids was introduced in [27,28], where the brittle crack propagation problem was regularized and recast as a minimization problem. In the phase-field approach, discontinuities are not introduced into the displacement field or geometrically described. Instead, a continuous field governed by a partial differential equation models cracks and their evolution. Crack propagation does not require evaluating stress intensity factors. This method naturally deals with crack nucleation, branching and coalesce result in a simple implementation. Its main drawback is its high computational cost. The crack zone is controlled by a regularization parameter. As this regularization parameter converges to zero, the phase field model converges to a discrete crack model.

Dealing computationally with the Kirchhoff–Love theory is challenging because second derivatives of the displacement field appear in the weak form, and therefore a Galerkin method requires C^1-continuous basis functions. This can be overcome by discretizing the director field or introducing rotational degrees of freedom [29–31], or by considering more elaborate variational formulations

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such as in discontinuous Galerkin methods [32,33]. Instead, here we focus on methods relying on smooth basis functions. Finite element methods with high order continuity have been proposed, either based on subdivision surfaces [13,34] or on isogeometric analysis [35–37]. The higher order continuity of the meshfree basis functions has also been exploited for this purpose [14,15], but since meshfree basis functions are defined in physical space, these methods were applied to simple geometries with a single parametric patch. Recently, nonlinear manifold learning techniques have been exploited to parametrize 2D sub-domains of a point-set surface, which are then used as parametric patches and glued together with a partition of unity [38,21]. Here, we combine this methodology with local maximum-entropy (LME) meshfree approximants [39,40,5] because of their smoothness, robustness, and relative ease of quadrature compared with other meshfree approximants.

The paper is organized as follows. Section 2 describes the representation of general surfaces represented by a set of scattered points [21]. In Section 3, we review the Kirchhoff–Love theory of thin shells. In Section 4, we introduce a phase-field model for fracture in thin shells. The Galerkin discretization is also presented in this section. In Section 5 we demonstrate the capabilities of the method through two numerical examples. Some concluding remarks are given in Section 6.

2. Numerical representation of the surfaces

To illustrate the method considered here for numerically representing surfaces defined by a set of scattered points, we refer to Fig. 1. As noted in [41], a fundamental difficulty in defining basis functions and performing calculations on a surface, as compared to open sub-sets in Euclidean space, is the absence in general of a single parametric domain. A simple example is the sphere, which does not admit a single singularity-free parametrization. Mesh-based methods, consisting of a collection of local parametrizations from the parent element to the physical elements, do not have any difficulty in this respect at the expense of reduced smoothness across the element boundaries or the need for special techniques to recover inter-element smoothness. In meshfree methods, such a natural parametric domain is not available, and the description of surfaces with a topology different to that of an open set in $\mathbb{R}^2$, such as a sphere (A) or a set of connected pipes (B), is a challenge. Even for surfaces homeomorphic to open two-dimensional sets, such as that depicted in (C), the geometric complexity can make it very difficult to produce well-behaved global parametrizations. For these reasons, the method we follow here proceeds in four steps: (1) We first partition the set of scattered points into subsets. (2) For each subset, the geometric structure of the surface is detected by dimensionality reduction methods and its points are embedded in 2D. (3) The 2D embedding then serves as a local parametric patch, and a local parametrization of the surface using smooth meshfree LME approximants is defined. (4) Finally, the different patches are glued together by means of a partition of unity.

Consider a smooth surface $\mathscr{M}$ embedded in $\mathbb{R}^3$ and represented by a set of (control) points $P = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_N\} \subset bopenR^3$. The goal is to numerically represent $\mathscr{M}$ from P and make computations on it. We consider another set $Q = \{\boldsymbol{Q}_1, \boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_M\} \subset \mathbb{R}^3$ with fewer point, typically a subset of P but not necessarily. We call the points of this set geometric markers. For simplicity, we will denote the points in P and its associated objects with a lower case subindex, e.g. $\boldsymbol{P}_a$, for $a = 1, 2, \ldots, N$, and the geometric markers in Q and its associated objects with an upper case subindex, e.g. $\boldsymbol{Q}_A$, for $A = 1, 2, \ldots, M$.

We partition these geometric markers into L groups. These L groups of geometric markers can be represented with index sets $\mathscr{I}_\kappa, \kappa = 1, \ldots, L$ with $\cup_{\kappa=1}^{L} \mathscr{I}_\kappa = \{1, 2, \ldots, M\}$ and $\mathscr{I}_\kappa \cap \mathscr{I}_\iota = \emptyset$ such that $\kappa \neq \iota$. As it will become clear below, there is a one-to-one correspondence between these groups of geometric markers and the local parameterizations of the surface, which here we refer as patches.

We consider a Shepard partition of unity associated with the geometric markers. Given a set of non-negative reals $\{\beta_A\}_{A=1,2,\ldots,M}$, we define the Shepard partition of unity with Gaussian weight associated to the set Q as

$$w_A(\boldsymbol{x}) = \frac{\exp(-\beta_A \mid \boldsymbol{x} - \boldsymbol{Q}_A|^2)}{\sum_{B=1}^{M} \exp(-\beta_B \mid \boldsymbol{x} - \boldsymbol{Q}_B|^2)}. \tag{1}$$

To obtain a coarser partition of unity representative of a partition, we aggregate the partition of unity functions as

$$\psi_\kappa(\boldsymbol{x}) = \sum_{A \in \mathscr{I}_\kappa} w_A(\boldsymbol{x}). \tag{2}$$

These functions form a partition of unity in $\mathbb{R}^D$, and consequently also in $\mathscr{M}$. We consider the index sets of all control points influencing each patch, $\mathscr{J}_\kappa$, with $\cup_{\kappa=1}^{L} \mathscr{J}_\kappa = \{1, 2, \ldots, N\}$, but now $\mathscr{J}_\kappa \cap \mathscr{J}_\iota \neq \emptyset$ due to the overlap between patch partition of unity functions. Roughly speaking, these sets are $\{a \mid \boldsymbol{P}_a \in \sup \psi_\kappa\}$, slightly enlarged so that the patch parameterization is smooth on the boundary of the support of ψ_κ.

For each patch, through a nonlinear dimensionality reduction technique applied to the set of control points $P_\kappa = \{\boldsymbol{P}_a\}_{a \in \mathscr{J}_\kappa} \subset \mathbb{R}^3$, we obtain a two-dimensional embedding of these points, represented by the set $\Xi_\kappa = \{\xi_a\}_{a \in \mathscr{J}_\kappa} \subset \mathbb{R}^2$. The two-dimensional region defined by these points is a convenient parametric space for the corresponding patch. It is important to note that the embedded points are in general unstructured, and that, although here $d = 2$, the methodology is applicable to higher dimensional embedded manifolds unlike mesh based techniques.

The patch parametrizations often need to be smooth, here because of the requirements of the Kirchhoff–Love theory. We consider here LME basis functions. See [39,40,38] for the LME

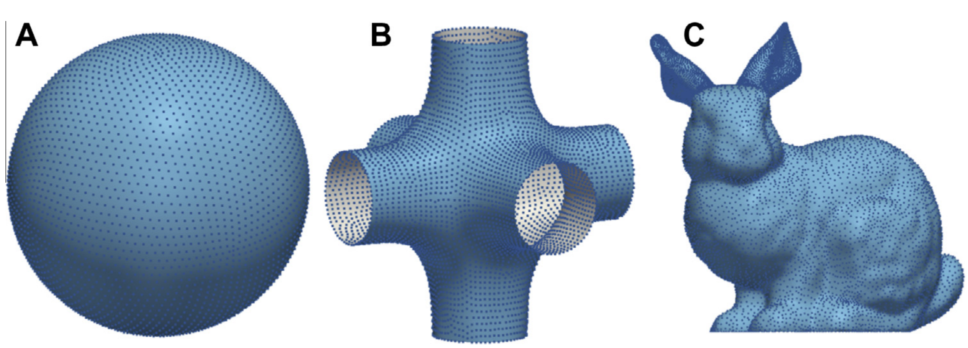

Fig. 1. Three point-set surfaces that require partitioning for different reasons: (A and B) for their non-trivial topology, and (C) for its complex geometry.