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A mass matrix formulation for cohesive surface elements

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ABSTRACT

A well-known method for modelling crack propagation in structural finite element analysis is the use of interface elements employing the theory of cohesive surfaces. However, the use of cohesive surfaces in explicit dynamics is problematic since they have zero mass and must initially be very stiff in order to avoid the introduction of artificial compliance. These properties lead to an often drastic reduction in the critical time step of the analysis. In this paper we use the bipenalty method to derive a mass matrix for a 2D cohesive surface interface element that does not add net physical mass to the overall system. This allows for cohesive surfaces with very high initial stiffness that have no effect on the critical time step of the analysis. Not only does this lead to a more robust and stable system, it also greatly simplifies the choice of parameters since there is no need to adjust the time step, and no need to limit the initial penalty stiffness according to time step stability considerations.

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1. Introduction

In finite element (FE) analysis the three most common techniques for the modelling of fracture and crack propagation in a dynamic setting are the element deletion method, the extended finite element method (XFEM), and inter-element crack methods [1]. Each of these approaches build upon standard FE formulations to include the effects of damage and crack propagation in some way. Element deletion is the simplest of the methods and the most widely used in commercial codes (e.g., ANSYS [2] and LS-DYNA [3]). It requires only an alteration of the constitutive relation of a failing element so that the stress in the element is reduced to zero for large strain, effectively removing certain elements as an analysis is carried out. However, its reliability with regards to the prediction of crack paths has been called into question [1]. Furthermore, crack paths and the details of crack growth are often highly mesh-dependent [4]. XFEM was first introduced by Belytschko and co-workers in 1999 to tackle crack propagation problems in elastostatics [5,6]. It uses shape function enrichment in order to introduce discontinuities within finite elements, which overcomes the high mesh dependence of previously existing techniques. This makes it an attractive option for accurately and efficiently predicting crack paths which are not known a priori [7], but has yet to achieve widespread adoption in commercial software.

Inter-element crack methods are a well-established group of techniques that explicitly model cracks on the boundaries of individual finite elements. This can be achieved either by adaptive remeshing or by the addition of interface elements at element boundaries possessing a specially designed traction–displacement relationship, an approach also referred to as the cohesive zone model. The theory of cohesive surfaces (also known as cohesive zones) was first introduced in the 1960s [8,9] but was not applied to dynamic crack propagation until the 1990s, with publications from Xu and Needleman [10], Camacho and Ortiz [11] and Repetto et al. [12] forming the basis for the present work. Each of these formulations introduces interface elements, or 'cohesive surfaces', into the FE continuum. A nonlinear traction–displacement relationship is then chosen that approximately represents the fracture characteristics of the material. Cracks are thus free to coalesce and propagate as a natural outcome of the simulation.

Using cohesive zone modelling for explicit dynamic analysis, however, leads to some unique challenges. Explicit solvers are much more efficient that implicit schemes per time step, but because they are conditionally stable the step size must be kept below the so-called critical time step, Δt_{crit}, in order to ensure stability. For the central difference method the critical time step is given by $\Delta t_{\text{crit}} = 2/\omega_{\max}$, where $\omega_{\max}$ is the maximum eigenfrequency of the system. The critical time step therefore depends on mesh size, as well as material properties. Elements with high stiffness or low mass decrease Δt_{crit}, leading to extra computational expense. Interface elements in a cohesive surface formulation must initially have very high stiffness so that they do not have any adverse effect on the simulation before damage has occurred; elements that are not stiff enough lead to 'artificial compliance' in the continuum [13,14]. In addition, they have no mass, since they have an initial volume of zero. These properties can lead to a drastic reduction in the critical time step that is required for stability.

Camacho and Ortiz [11] avoid this problem by introducing cohesive surfaces only at the onset of damage, but this requires

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alterations to the FE discretisation (and thus to the computer memory requirements) as cracks propagate. Ortiz and Pandolfi [15] also select a cohesive law without an initial elastic region because this would place "stringent restrictions" on the stable time step. Espinosa and Zavattieri [13] use a large initial stiffness, but it is acknowledged by the same authors that a large penalty will have a significant impact on the critical time step, and as a result, the time step calculation includes an additional limitation in that it must take into account the cohesive surfaces as well as continuum elements. Because of this, a subcycling time integration routine is built into the formulation, adding undesirable complexity to the solution algorithm. This is deemed necessary because, as noted by Song et al., the original cohesive surface formulation developed by Xu and Needleman "induces artificial compliance due to the elasticity of the intrinsic cohesive law" [14].

Interface elements by their nature introduce large eigenvalues into the FE system; since the critical time step is inversely proportional to the maximum eigenvalue this has a detrimental effect on the critical time step. The standard analysis states that this is due to the high initial stiffness of the cohesive surface elements. However, eigenvalues may be decreased not only by decreasing the stiffness of an element, but also by adding mass. Recently, an extension of the traditional penalty method—referred to here as the bipenalty method—has been proposed that includes a mass penalty matrix alongside standard stiffness penalties in the formulation [16–20]. In the present work, we use the bipenalty method to provide a mass matrix for a simple cohesive surface formulation. No net physical mass is added to the system; the sum of all elements in the interface mass matrix is zero. The inclusion of the mass matrix, however, does allow for control over the eigenvalues introduced by the interface elements, and therefore control over the effect that the elements have on the critical time step. By providing a mass matrix formulation alongside the traditional stiffness penalties, the introduced eigenvalues can be controlled even when very a very large initial stiffness is used, so that interface elements and, by extension, cohesive surfaces can be used in explicit dynamics without having to reduce the critical time step.

2. Element formulation

We assume that initially we have a structural system, discretised in space by the FE method, of the form

$$\mathbf{M}\,\mathbf{u} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the assembled mass and stiffness matrix for the continuum elements, $\mathbf{u}$ is the displacement vector, $\mathbf{f}$ the external force vector, and dot notation is used to indicate time derivatives; structural damping is neglected. A bipenalty formulation results in a system of equations of the form

$$(\mathbf{M} + \mathbf{M}^{\mathrm{P}})\,\mathbf{u} + (\mathbf{K} + \mathbf{K}^{\mathrm{P}})\mathbf{u} = \mathbf{f} \tag{2}$$

where $\mathbf{M}^{\mathrm{P}}$ and $\mathbf{K}^{\mathrm{P}}$ are mass and stiffness penalty matrices, which for a system containing cohesive surfaces are assembled from the interface element mass and stiffness matrices, which are to be derived in this section.

The critical time step for the system is given by

$$\Delta t_{\mathrm{crit}} = \frac{2}{\omega_{\max}} \tag{3}$$

where $\omega_{\max}$ is the maximum eigenfrequency of the system. Eigenvalues are related to eigenfrequencies by $\lambda_i = \omega_i^2$ and the maximum eigenvalue is $\lambda_{\max}$. The eigenvalues can be determined by solving the generalised eigenvalue problem for the system. In the case where $\mathbf{K}^{\mathrm{P}} = R\mathbf{M}^{\mathrm{P}}$ (with R a scalar) it has been shown that the maximum eigenvalue $\lambda_{\max}$ of the penalised system (2) will not exceed the maximum eigenvalue $\lambda_{\max}^{\mathrm{UP}}$ of the unpenalised system (1) for the case where $R \leqslant \lambda_{\max}^{\mathrm{UP}}$ [20,21]. Thus, the critical time step Δt_{crit} is not decreased by the addition of the interface elements for $R \leqslant \lambda_{\max}^{\mathrm{UP}}$.

We will now present a standard interface element stiffness matrix formulation, followed by the corresponding mass matrix formulation, and show that under reasonable assumptions, $\mathbf{K}^{\mathrm{P}} = R\mathbf{M}^{\mathrm{P}}$ (and therefore that the above analysis holds for this bipenalty cohesive surface formulation).

2.1. Element stiffness matrix

The interface element formulation is based on the work of Schellekens [22,23], who derives a 4-noded 2D line interface element with an initial volume of zero (see Fig. 1). The stress is defined by normal and tangential tractions across the interface and the stiffness of the element is controlled by user-defined parameters that describe the constitutive behaviour.

We now consider this 4-noded line interface. Each node has two displacement degrees of freedom (DOF), giving an element nodal displacement vector

$$\mathbf{d} = \left[d_{\mathrm{n}}^1, d_{\mathrm{n}}^2, d_{\mathrm{n}}^3, d_{\mathrm{n}}^4, d_{\mathrm{t}}^1, d_{\mathrm{t}}^2, d_{\mathrm{t}}^3, d_{\mathrm{t}}^4\right]^T \tag{4}$$

where n and t denote the directions normal and tangential to the interface, respectively, and superscripts indicate the node numbers as shown in Fig. 1. The relationship between nodal displacements $\mathbf{d}$ and relative displacements $\boldsymbol{\delta} = [\delta_{\mathrm{n}}, \delta_{\mathrm{t}}]^T$ is given by

$$\boldsymbol{\delta} = \mathbf{B}\mathbf{d} \tag{5}$$

where

$$\mathbf{B} = \begin{bmatrix} -\mathbf{n} & \mathbf{n} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{n} & \mathbf{n} \end{bmatrix}$$

and $\mathbf{n}$ are the interpolation polynomials $\mathbf{n} = [N_1, N_2]$. For arbitrarily orientated elements, the matrix $\mathbf{B}$ should be transformed to the local tangential co-ordinate system of the node set.

We now introduce a matrix $\mathbf{D}_{\mathrm{s}}$ describing the constitutive traction–displacement relation, so that

$$\mathbf{t} = \mathbf{D}_{\mathrm{s}}\boldsymbol{\delta} \tag{7}$$

where $\mathbf{t} = [t_{\mathrm{n}}, t_{\mathrm{t}}]^T$ is the traction vector for the element (units N/m^2) and $\mathbf{D}_{\mathrm{s}}$ is a constitutive matrix of the form

$$\mathbf{D}_{\mathrm{s}} = \begin{bmatrix} k_{\mathrm{n}} & 0 \\ 0 & k_{\mathrm{t}} \end{bmatrix}$$

The values k_{n} and k_{t} (units N/m^3) represent the 'stiffness' of the interface in the normal and tangential directions, although a more accurate description is stiffness per unit area. It is these values that function as the stiffness penalty parameters for the interface. In the present work we assume that both parameters are equal so that $k_{\mathrm{n}} = k_{\mathrm{t}} = \alpha_{\mathrm{s}}$ and $\mathbf{D}_{\mathrm{s}} = \alpha_{\mathrm{s}}\mathbf{I}$. We postpone until Section 2.3 a discussion of how these constitutive relations may change over time (due to damage).

The stiffness matrix $\mathbf{K}_{\mathrm{e}}$ can now be obtained by minimisation of the total potential energy. The internal work done in the element is

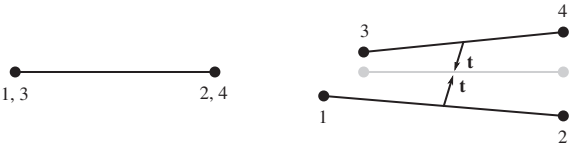

Fig. 1. Line interface element in an initial (left) and deformed (right) configuration.

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