Theoretical and Applied Fracture Mechanics 69 (2014) 118–125

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

A meshless sub-region radial point interpolation method for accurate calculation of crack tip fields

Xiaoying Zhuang [a,c], Yongchang Cai [a,*], Charles Augarde [b]

[a] National Key Laboratory for Disaster Reduction and Prevention, Tongji University, 1239 Siping Road, Shanghai 200092, China
[b] School of Engineering and Computing Sciences, Durham University, South Road, Durham DH1 3LE, UK
[c] Institute of Structural Mechanics, Bauhaus-Universität, Weimar 99423, Germany

ARTICLE INFO

Article history:
Available online 16 December 2013

Keywords:
Crack tip
Meshless
Meshfree
Mixed variational principle
RPIM

ABSTRACT

A new meshless sub-region radial point interpolation method (MS-RPIM) is proposed for linear elastic fracture mechanics. The Williams expansions of stress field for mode I/II crack is used as the trial functions in crack tip region, the meshless radial point interpolation is used for the rest of domain, and a mixed variational principle is used for discretisation. In contrast to existing meshless formulations, the present MS-RPIM requires only very few nodes around the crack tip to obtain smooth stress and accurate results and the SIFs can be directly obtained as part of the solution and no additional effort via post-processing.

1. Introduction

The accurate analysis of crack tip fields is of vital importance for the safety assessment and life prediction of cracked engineering structures and materials. Over the past three decades, a wide range of numerical methods have been proposed for fracture modelling. The finite element method (FEM) using quarter-point for standard elements, singular crack tip elements, enriched elements, and hybrid elements [1–5] can be applied for fracture modelling with quite good accuracy. For static cracks, the FEM remains a dominant numerical tool. However, the method finds difficulties in modelling crack propagation due to the element topology that needs to be updated during crack propagation. More recently, the development of novel numerical methods has attracted much interest for researchers in computational mechanics particularly in the area of meshless methods, which refers to a group of numerical methods requiring no preexisting mesh for the construction of the field approximation. They are particularly suitable for fracture modelling since there is no entanglement problem with large deformations of the mesh requiring updating or remeshing to accommodate the changing geometry of a crack. Some of the prominent methods for crack analysis are the generalised finite element method (GFEM), the extended finite element method (XFEM), smoothed FEM and non-uniform B-spline based FEM [6–8,44]. These methods together with meshless methods fall generally into the family of partition of unity methods.

Recently, much effort has been directed towards the application of meshless methods to crack problems to overcome the difficulties in traditional numerical methods such as meshless methods for dynamic problem, 2D fracture modelling [9–18], 3D fracture modelling [19,38–41], fluid structure integration [42], cohesive cracks [29–33], concrete fragmentations [34–37] and shell analysis [43]. Despite clear general progress with these methods, there are still some technical issues in their application to fracture problems, for instance, it is often awkward and an expensive task to refine the nodal arrangement near the crack tip in order to increase the solution accuracy, since the stress results tend to be oscillatory near the crack tip. The incorporation of singular functions associated with linear elastic fracture in meshless methods reduces stress oscillations and increases accuracy of stress intensity factor (SIF) significantly [12,16]. However, introducing such an enriched basis in meshless approximations can lead to ill-conditioning of the global stiffness matrix, and special treatments [12,17] have to be used to alleviate this problem. Thirdly, many meshless methods employ the *J*-integral or contour integral scheme for the calculation of SIF, which is performed as a post-processing step applied to the stress results, such as in the formulations using the FEM described in [15–19] and partition of unity enriched boundary element method (PU-BEM) [20,21]. This is unlike the case with the isoparametric FEM or sub-region mixed variational principle based FEM where the SIF can be directly obtained as part of the solution [3–5].

To address the above issues, a new meshless method is proposed in this paper which can be classified as a mixed sub-region radial point interpolation method (MS-RPIM) for analyzing crack

* Corresponding author. Tel.: +86 21 65985014; fax: +86 21 65985140.
E-mail address: yccai@tongji.edu.cn (Y. Cai).

0167-8442/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.tafmec.2013.12.003

tip fields. In this method, Williams expansion of the stress field of mode I/II crack [22] is used as the trial functions in the region near the crack tip, the meshless RPIM [23,24] is used for the region far from the crack tip, and a mixed variational principle is used to discretise the governing equations [3]. The present MS-RPIM preserves the advantages of meshless methods where the entanglement of finite element topology is removed, and has further positive features such as a simple formulation for numerical implementation. In contrast to existing meshless formulations for fracture modelling, it has the following advantages. Firstly, only a very few nodes around the crack tip are required to obtain smooth stress results and accurate SIFs. Secondly, solution accuracy and stability are much better than meshless methods using implicit enrichment and it is free from the ill-conditioning problem which affects the global stiffness matrix using explicit enrichment. Finally, the SIFs can be directly obtained as part of the solution; there is no additional effort required to calculate the SIF results via post-processing. The rest of the paper is organised as follows. Section 2 covers the field interpolation used in the MS-RPIM, which is then described in detail in Section 3 including a description of the mixed variational formulation used for discretisation. Finally, Section 4 contains verification examples to show the performance of the method.

2. Point interpolation based on radial basis function

For the convenience of the following discussion, in this section we will briefly describe the field interpolation using the RPIM, used for the stress analysis. The RPIM was originally proposed in [23] and has been recently used for fracture modelling in [16,24]. We confine the present study to 2D linear elastic fracture mechanics, with the fundamental field variables being displacements. Consider a problem domain Ω of arbitrary shape discretised by a set of scattered nodes $\{\mathbf{x}_i\}$ as shown in Fig. 1. For a given point $\mathbf{x}$ in Ω, there are n distributed nodes in the influence domain Ω_x of point $\mathbf{x}$. Considering one of the two freedoms at a node, the nodal function value is u_i at node $\mathbf{x}_i$. The RPIM is used to construct the approximation function $u(\mathbf{x})$ of the point $\mathbf{x}$ using radial basis functions $B_i(\mathbf{x})$ and polynomial basis functions $P_i(\mathbf{x})$ having m terms

$$u(\mathbf{x}) = \sum_{i=1}^{n} B_i(\mathbf{x})a_i + \sum_{j=1}^{m} P_j(\mathbf{x})b_j = \mathbf{B}^{\mathrm{T}}(\mathbf{x})\mathbf{a} + \mathbf{P}^{\mathrm{T}}(\mathbf{x})\mathbf{b} \tag{1}$$

where the vectors are defined as

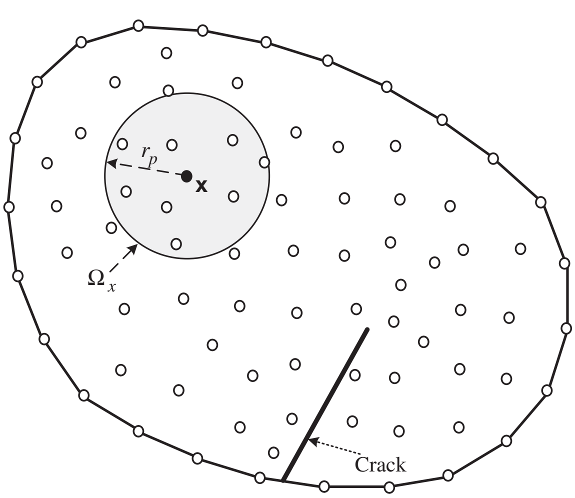

Fig. 1. The messhless model for an arbitrary analysis domain.

$$\begin{aligned} \mathbf{a}^{\mathrm{T}} &= [a_1, a_2, \ldots, a_n] \\ \mathbf{b}^{\mathrm{T}} &= [b_1, b_2, \ldots, b_m] \\ \mathbf{B}^{\mathrm{T}}(\mathbf{x}) &= [B_1(\mathbf{x}), B_2(\mathbf{x}), \ldots, B_n(\mathbf{x})] \\ \mathbf{P}^{\mathrm{T}}(\mathbf{x}) &= [P_1(\mathbf{x}), P_2(\mathbf{x}), \ldots, P_m(\mathbf{x})] \end{aligned} \tag{2}$$

By enforcing the interpolation to pass through all n nodes within the influence domain Ω_x, the coefficients a_i and b_i in Eq. (1) can be determined, and the RPIM interpolation can be expressed as

$$u(\mathbf{x}) = \begin{bmatrix} \mathbf{B}^{\mathrm{T}}(\mathbf{x}) & \mathbf{P}^{\mathrm{T}}(\mathbf{x}) \end{bmatrix} \mathbf{G}^{-1} \begin{Bmatrix} \mathbf{u} \\ \mathbf{0} \end{Bmatrix} = \mathbf{\Phi u} = \sum_{i=1}^{n} \phi_i(\mathbf{x}) u_i \tag{3}$$

where $\phi_i(\mathbf{x})$ is the RPIM shape function, $\mathbf{u}$ is the vector of nodal values where

$$\mathbf{u}^{\mathrm{T}} = [u_1, u_2, \ldots, u_n] \tag{4}$$

and $\mathbf{G}$ is

$$\mathbf{G} = \begin{bmatrix} \mathbf{B}_n & \mathbf{P}_m \\ \mathbf{P}_m^{\mathrm{T}} & \mathbf{0} \end{bmatrix} \tag{5}$$

in which

$$\mathbf{B}_n = \begin{bmatrix} B_1(\mathbf{x}_1) & B_2(\mathbf{x}_1) & \cdots & B_n(\mathbf{x}_1) \\ B_1(\mathbf{x}_2) & B_2(\mathbf{x}_2) & \cdots & B_n(\mathbf{x}_2) \\ \vdots & \vdots & \vdots & \vdots \\ B_1(\mathbf{x}_n) & B_2(\mathbf{x}_n) & \cdots & B_n(\mathbf{x}_n) \end{bmatrix}_{n \times n} \tag{6}$$

and

$$\mathbf{P}_m = \begin{bmatrix} P_1(\mathbf{x}_1) & P_2(\mathbf{x}_1) & \cdots & P_m(\mathbf{x}_1) \\ P_1(\mathbf{x}_2) & P_2(\mathbf{x}_2) & \cdots & P_m(\mathbf{x}_2) \\ \vdots & \vdots & \vdots & \vdots \\ P_1(\mathbf{x}_n) & P_2(\mathbf{x}_n) & \cdots & P_m(\mathbf{x}_n) \end{bmatrix}_{n \times m} \tag{7}$$

It has been proven in [24] that the RPIM shape functions $\phi_i(\mathbf{x})$ in Eq. (3) possess the Kronecker delta property. Hence, essential boundary conditions in the RPIM method can be directly applied as in the FEM. Here, a linear polynomial basis function

$$\mathbf{P}^{\mathrm{T}}(\mathbf{x}) = [1, x, y] \tag{8}$$

and Gaussian type radial basis function

$$B_i(\mathbf{x}) = \exp\left[-0.31\left(\frac{r_i}{r_p}\right)^2\right] \tag{9}$$

are used in the present study. In Eq. (9), r_p is the radius of the influence domain Ω_x of point $\mathbf{x}$, and r_i is a distance between interpolating point $\mathbf{x}$ and the node $\mathbf{x}_i$ where

$$r_i^2 = (x - x_i)^2 + (y - y_i)^2 \tag{10}$$

To capture the displacement discontinuity due to the existence of crack as shown in Fig. 1, the visibility criterion [12] is used where a point of interest and the nodes supporting that point severed by a crack is not associated in the interpolation. For the determination of the radius r_p at point $\mathbf{x}$, a radius d_i of the support domain for an arbitrary node $\mathbf{x}_i$ in domain Ω is firstly defined as

$$d_i = \alpha \cdot c_i \tag{11}$$

where c_i is set as the distance to the fifth nearest neighbour node near node $\mathbf{x}_i$, α is a coefficient and here $\alpha = 2.7$ for the nodes near the crack and at the boundary, and $\alpha = 2.0$ for all other nodes. This is due to the fact that when the visibility criterion is used to exclude the nodes cut by the crack, or when an integration point or a sampling point is close to the boundary, less nodes are included as supporting nodes. This may lead to an ill conditioned problem