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Triaxial stress state of cylindrical openings for rocks modeled by elastoplasticity and strength criterion

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ABSTRACT

An analytical solution for calculating the triaxial stress state around a cylindrical opening in an elastoplastic cohesive medium is developed. Magnitude of the slip zones around a cylindrical opening in crack-weakened rock masses is considered by modifying the existing strength criterion for rocks of different types. The disturbance coefficient, the geological strength index and the other strength parameters are also investigated.

1. Introduction

The deformation and failure of rock masses around circular openings are examined by finding the corresponding stress state and the plastic zones. In practice, the openings may represent boreholes, and tunnels [1–6]. Previous methods have entailed numerical schemes such as FEM, BEM etc. Theoretical models made use of elasticity and elastoplasticity. Elastoplastic solutions for circular openings [1–14] have involved the application of a hydrostatic stress σ_v at infinity. The medium is continuous and the plane strain is invoked. Axisymmetric elastoplastic or elasto-perfect-plastic plane strain analyses have made use of the linear Mohr–Coulomb (M–C) criterion and the non-linear Hoek–Brown criterion. They cannot account for the triaxial stress state, and is applied only for situations with two principal stresses. The intermediate principal stress was not considered.

Rock material encountered in engineering practice is inhomogeneous and discontinuous containing joints, cracks, etc. Imperfections of crack-weakened rock masses should be accounted for because they can affect the loading supporting ability of the rocks. Effects of cracks on slip zones around a circular underground opening has been described in [15]. The detailed procedure can be found in [16]. In both these studies, it is assumed that the medium containing the circular underground opening is characterized by the Mohr–Coulomb criterion or the Hoek–Brown criterion. Slip zones around the circular openings were determined from the Mohr–Coulomb criterion or the Hoek–Brown criterion [17]. However, the intermediate principal stress were not taken into account.

The theoretical analyses from a 2D yield criterion are not always appropriate when triaxiality takes precedent such as the triaxial compression (CTC) stress conditions that are encountered around a circular underground opening. This effect has been studied by many past researchers. For instance, Mogi's persistent effort revealed that rock strength varied with the intermediate principal stress (σ_2), which was quite different from what had been predicted from the conventional linear Mohr–Coulomb theory [18]. Further studies [19–23] showed that the intermediate principal stress gives rise to two zones: (1) strength of rock σ_1 increases with an increase in the intermediate principal stress from $\sigma_2 = \sigma_3$ to a maximum value (reach the peak strength when $\sigma_2 = \sigma_2'$); (2) strength of rock σ_1 reduces with further increasing of the intermediate principal stress from $\sigma_2 = \sigma_2'$ to $\sigma_2 = \sigma_1$. It was shown that varying σ_2 only, while keeping the other principal stresses (σ_1 and σ_3) unchanged, could lead to rock failure [22,23].

2. The modified strength criterion

The modified strength criterion has the following characteristics [24]:

(1) It reflects the characteristics of rocks with different tensile and compressive strengths, while effects of hydrostatic pressure and the intermediate principal stress are also included.
(2) Use of the modified strength criterion together with elastoplasticity provided a closed form solution that entails the pertinent material parameters for rocks.

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(3) Results are shown to be consistent with tests data in the literature for different rock types under various stress states.

Based on the triaxial compression tests and plane strain test data for rocks, a modified strength criterion has been obtained [24]:

$$F = 0, \quad \text{when } F \geqslant F' \tag{1}$$

and

$$F' = 0 \quad \text{when } F \leqslant F' \tag{2}$$

where

$$F = \sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3) - \sigma_c\left[\frac{m}{(1+b)\sigma_c}(b\sigma_2+\sigma_3) + s\right]^{1/2} \tag{3}$$

and

$$F' = \frac{1}{1+b}(b\sigma_2 + \sigma_1) - \sigma_3 - \sigma_c\left[\frac{m}{\sigma_c}\sigma_3 + s\right]^{1/2} \tag{4}$$

where σ_c is an uniaxial compressive strength (macroscopic) of an intact rock material, σ_1 is the maximum principal stress at failure, σ_2 is the intermediate principal stress at failure, σ_3 is the minimum principal stress at failure, which can be microscopic in the rock material at the prospective site of yielding. m and s are the material parameter as same as those in the Hoek–Brown criterion. b is the intermediate stress parameter. The parameter b is also a parameter of the strength criterion which varies from 0 to 1. The magnitude of m and s depends on the geological strength index (GSI) which characterizes the quality of the rock masses. GSI depends on its structure and the surface condition of the joints. The strength parameters m and s are defined in [25]

$$\frac{m}{m_i} = \exp\left(\frac{\text{GSI} - 100}{28 - 14D}\right) \tag{5}$$

$$s = \exp\left(\frac{\text{GSI} - 100}{9 - 3D}\right) \tag{6}$$

where D is a disturbance coefficient which varies from 0.0 for the undisturbed in situ rock masses to 1.0 for very disturbed rock masses, m_i is the value of m for intact rock and can be obtained from experiments. The parameter m_i varies from 4 for very fine weak rock like claystone to 33 for coarse igneous light-colored rock like granite. If there are no test data available, approximate values for five types of rocks are available [26]:

(a) $m_i \approx 7$ for carbonate rocks with well developed crystal cleavage (dolomite, limestone and marble);
(b) $m_i \approx 10$ for lithified argillaceous rocks (mudstone, siltstone shale and slate);
(c) $m_i \approx 15$ for arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone and quartzite);
(d) $m_i \approx 17$ for fine-grained polymineralic igneous crystalline rocks (andesite, dolerite, diabase and rhyolite);
(e) $m_i \approx 25$ for coarse-grained polymineralic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite and quartz-diorite).

When $b = 0$, the modified strength criterion is simplified to Hoek–Brown criterion, a single-shear strength criterion, which forms the lower bound. When $b = 1$, the modified strength criterion is simplified to a non-linear twin-shear strength criterion that forms the upper bound. All the strength criterion ranging from the Hoek–Brown criterion (the lower bound) to a non-linear twin-shear criterion (the upper bound) and series of criteria ranging these two bounds may be introduced by the modified strength criterion.

It should be mentioned that the conventional yield criterion of the type used in Eqs. (1)–(4) is based on the assumption that the local state of stress (triaxial or otherwise) as determined for a continuum element would yield as a combination, the threshold of which is assume to correspond to some equivalent uniaxial stress state that is determinable from macroscopic tests (compression or tension). The local stresses in the tests can presumably be associated with the slip planes of the rocks and hence be microscopic, although continuum elastoplastic equations were used to find the local stresses. This entails a conceptual difficulty in that *the macro theory stresses were assumed to correspond to the actual micro stresses related to the slip planes.* Anomalies associated with the transition of micro to macro quantities such as stresses and energy densities have been discussed [27,28] and they are particularly relevant to rock mechanics where macroscopic theories are used to explain microscopic phenomena.

3. Stress state around circular openings in an elastoplastic medium

Fig. 1 shows a circular opening located in an elastoplastic medium subjected to a far field hydrostatic pressure $\sigma_v = \sigma_h$. Here, r_0 is the initial radius of the underground opening; p_0 is an internal pressure in the cylinder; R is the radius of the interface between yielded (plastic) and unyielded (elastic) regions; r and θ are the cylindrical co-ordinates of a given location, whereas σ_r and σ_θ are the corresponding radial and tangential stresses, respectively. They shall be referred to as elastic and plastic regions. Their separation is known as the elastic–plastic boundary.

It is assumed that the medium is elastic–perfectly plastic. Under a plane strain condition,

$\varepsilon_z = 0$ (ε_z is the strain along the axis of the circular opening), the induced stress along the longitudinal axis becomes

$$\sigma_z = \sigma_a + \mu(\Delta\sigma_r + \Delta\sigma_\theta) \tag{7}$$

where μ is Poisson's ratio, σ_a is the in situ stress along the cylinder axis, $\Delta\sigma_r = \sigma_r - \sigma_v$ and $\Delta\sigma_\theta = \sigma_\theta - \sigma_v$ are variations of the radial and tangential stress, respectively.

Eq. (2) can be rewritten as

$$\sigma_z = \sigma_a - 2\mu\sigma_v + \mu(\sigma_r + \sigma_\theta) \tag{8}$$

If a circular opening is subjected to an external hydrostatic pressure, $\sigma_v = \sigma_h = \sigma_a$, the above expression can be expressed as [4]

$$\sigma_z = (1 - 2\mu)\sigma_v + \mu(\sigma_r + \sigma_\theta) \tag{9}$$

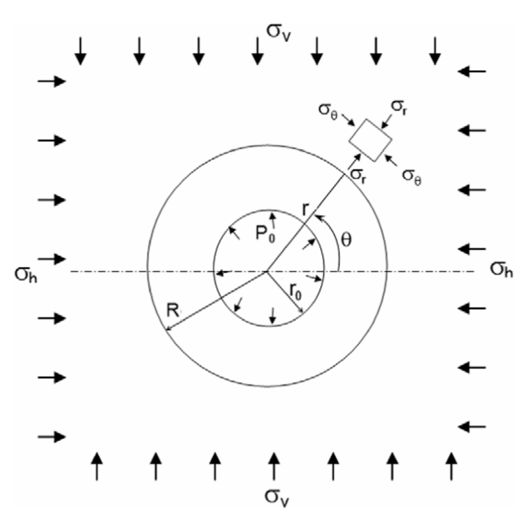

Fig. 1. A circular opening subjected to an external hydrostatic pressure $\sigma_h = \sigma_v$ and an internal pressure p_0; the natural stress along the cylinder axis (not shown) is σ_a.