Theoretical and Applied Fracture Mechanics 50 (2008) 226–234

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Propagation of waves in micropolar viscoelastic generalized thermoelastic solids having interficial imperfections

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ARTICLE INFO

Article history:
Available online 6 August 2008

Keywords:
Micropolar viscoelastic generalized thermoelastic solid
Normal force stiffness
Transverse force stiffness
Transverse couple stiffness
Thermal contact conductance
Perfect bonding
Amplitude ratios

ABSTRACT

The boundary conditions at an imperfect interface between two homogeneous, isotropic micropolar viscoelastic generalized thermoelastic half-spaces of different micropolar, thermal and viscous properties are solved for reflection and transmission coefficients. The expressions for the reflection and transmission coefficients which are the ratios of the amplitude of reflected and transmitted waves to the angle of incident wave are obtained for Lord-Shulman (L-S theory) theory of thermoelasticity and deduced for normal force stiffness, transverse force stiffness, transverse couple stiffness, thermal contact conductance and perfect bonding. Viscous and stiffness effects on these amplitude ratios with angle of incidence have been shown graphically. Some special cases have been deduced.

1. Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the elastomer and polymer type, e.g. polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves i.e. for the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. This influence of microstructure results in the development of new type of waves, not found in the classical theory of elasticity. Metals, polymers, composites, soils, rocks, concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a microstructure. Developed in [1,2] is the linear theory of micropolar elasticity. The linear theory of micropolar viscoelasticity has been developed [3,4]. They discussed the propagation conditions and growth equations which govern the propagation of waves in micropolar viscoelasticity.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continual to include thermal effects [5,6] and is known as micropolar coupled thermoelasticity. Presented in [7] is the generalized micropolar thermoelasticity by using the work in [9]. Developed in [10] is a heat-flux dependent theory of micropolar thermoelasticity.

An actual interface between two elastic solids is much more complicated and has physical properties different from those of the substrates. There are two classical elastic boundary conditions for solid/solid interface. One boundary condition for welded interface and other is slip boundary condition. A generalization of this concept is that of an imperfectly bonded interface for which displacement across a surface need not be continuous.

Imperfect bonding considered in the present investigation is to mean that the stress components are continuous and small displacement field is not. The small vector difference in the displacement is assumed to depend linearly on the traction vector. Significant work has been done to describe the physical conditions on the interface by different mechanical boundary conditions by different investigators. Notable among them are the works

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doi:10.1016/j.tafmec.2008.07.010

[11–19]. Recently various authors have used the imperfect conditions at an interface to study various types of problems[20–24].

Discussed in [25] is the wave propagation in a micropolar viscoelastic generalized thermoelastic solid and in [26] the harmonic waves in thermoviscoelastic solids. Presented in [27] is the reflection and refraction of micropolar magneto-thermoviscoelastic waves at the interface between two micropolar viscoelastic media. Also studied in [28] is the reflection and refraction of thermoelastic plane waves using the imperfect conditions at an interface between two thermoelastic media without energy dissipation. Recently, investigated in [29] is the reflection of wave at viscoelastic-micropolar elastic interface. To be presented is the reflection and transmission of plane waves between two micropolar viscoelastic generalized thermoelastic half-spaces of different micropolar, thermal and viscous properties and deduced the different cases.

2. Basic equations

Following the works in [3,8], the constitutive relations and equation of motion in micropolar viscoelastic generalized thermoelastic solid with one relaxation time in absence of body forces and body couples are given by

$$t_{kl} = \lambda_1 u_{r,r}\delta_{kl} + \mu_1(u_{k,l} + u_{l,k}) + K_1(u_{l,k} - \varepsilon_{klr}\phi_r) - \nu T\delta_{ij}, \tag{1}$$

$$m_{kl} = \alpha_1\phi_{r,r}\delta_{kl} + \beta_1\phi_{k,l} + \gamma_1\phi_{l,k} \quad (i,j,k,l = 1,2,3), \tag{2}$$

$$(c_1^2 + c_3^2)\nabla(\nabla\cdot\vec{u}) - (c_2^2 + c_3^2)\nabla\times(\nabla\times\vec{u}) + c_3^2\nabla\times\vec{\phi} - \bar{\nu}\nabla T = \frac{\partial^2\vec{u}}{\partial t^2}, \tag{3}$$

$$(c_4^2 + c_5^2)\nabla(\nabla\cdot\vec{\phi}) - c_4^2\nabla\times(\nabla\times\vec{\phi}) + \omega_0^2\nabla\times\vec{u} - 2\omega_0^2\vec{\phi} = \frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{4}$$

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla\cdot\vec{u}), \tag{5}$$

where

$$c_1^2 = \frac{\lambda_1 + 2\mu_1}{\rho},\quad c_2^2 = \frac{\mu_1}{\rho},\quad c_3^2 = \frac{K_1}{\rho},\quad c_4^2 = \frac{\gamma_1}{\rho j},\quad c_5^2 = \frac{\alpha_1 + \beta_1}{\rho j},$$
$$\omega_0^2 = \frac{K_1}{\rho j},\quad \bar{\nu} = \frac{\nu}{\rho} \tag{6}$$

and

$$\lambda_1 = \lambda + \frac{\partial}{\partial t}\lambda_v,\quad \mu_1 = \mu + \frac{\partial}{\partial t}\mu_v,\quad K_1 = K + \frac{\partial}{\partial t}K_v,\quad \alpha_1 = \alpha + \frac{\partial}{\partial t}\alpha_v,$$
$$\beta_1 = \beta + \frac{\partial}{\partial t}\beta_v,\quad \gamma_1 = \gamma + \frac{\partial}{\partial t}\gamma_v, \tag{7}$$

where λ, μ, K, α, β, γ, λ_v, μ_v, K_v, α_v, β_v, γ_v-material constants, ρ-density, T-the temperature distribution, T_0-reference temperature, $\vec{u}$ – displacement vector, $\vec{\phi}$-microrotation vector, j-microinteria, $\nu = (3\lambda_1 + 2\mu_1 + K_1)\alpha_t$, α_t-coefficient of linear thermal expansion, C^*-specific heat at constant strain, K^*-thermal conductivity, ε_{klr}-alternate tensor, t_{kl}-components of force stress tensor, m_{kl}-components of couple stress tensor,τ_0-the thermal relaxation time, δ_{ij}-Kronecker delta.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z},\quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

The necessary and sufficient conditions for the internal energy to be non-negative, as given by Eringen [3,5] are:

$$0 \leqslant 3\lambda + 2\mu + K,\quad 0 \leqslant \mu,\quad 0 \leqslant K,\quad 0 \leqslant 3\alpha + 2\gamma,$$
$$-\gamma \leqslant \beta \leqslant \gamma,\quad 0 \leqslant \gamma, \tag{8}$$

$$0 \leqslant 3\lambda_v + 2\mu_v + K_v,\quad 0 \leqslant \mu_v,\quad 0 \leqslant K_v,\quad 0 \leqslant 3\alpha_v + 2\gamma_v,$$
$$-\gamma_v \leqslant \beta_v \leqslant \gamma_v,\quad 0 \leqslant \gamma_v. \tag{9}$$

3. Formulation and solution of the problem

Consider two homogeneous, isotropic micropolar viscoelastic thermally conducting half-spaces being in contact with each other at the plane surface which we designate as the plane $z = 0$ of rectangular cartesian co-ordinate system OXYZ. Consider plane harmonic waves in xz-plane with wave front parallel to y-axis and all the field variables depend only on x, z, t.

By Helmholtz representation of a vector into scalar and vector potentials, it can be written that

$$\vec{u} = \nabla q + \nabla\times\vec{U},\quad \nabla\cdot\vec{U} = 0, \tag{10}$$

$$\vec{\phi} = \nabla\xi + \nabla\times\vec{\Phi},\quad \nabla\cdot\vec{\Phi} = 0. \tag{11}$$

Substituting Eqs. (10) and (11) in Eqs. (3)–(5), there results

$$(c_1^2 + c_3^2)\nabla^2 q = \frac{\partial^2 q}{\partial t^2} + \bar{\nu}T, \tag{12}$$

$$(c_4^2 + c_5^2)\nabla^2\xi - 2\omega_0^2\xi = \frac{\partial^2\xi}{\partial t^2}, \tag{13}$$

$$(c_2^2 + c_3^2)\nabla^2\vec{U} + c_3^2\nabla\times\vec{\Phi} = \frac{\partial^2\vec{U}}{\partial t^2}, \tag{14}$$

$$c_4^2\nabla^2\vec{\Phi} - 2\omega_0^2\vec{\Phi} + \omega_0^2\nabla\times\vec{U} = \frac{\partial^2\vec{\Phi}}{\partial t^2}, \tag{15}$$

$$\left[K^*\nabla^2 - \rho C^*\frac{\partial}{\partial t}\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right]T = \nu T_0\left[\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right]\nabla^2 q. \tag{16}$$

It can be noticed that Eqs. (12) and (16) are in scalar potentials q and T while Eqs. (14) and (15) constitute the coupled system in vector potentials $\vec{U}$ and $\vec{\Phi}$ and 13 is uncoupled.

For the two dimensional problem in xz-plane, the components of displacement and microrotation are given by

$$\vec{u} = (u_1, 0, u_3), \tag{17}$$

$$\vec{\phi} = (0, \phi_2, 0). \tag{18}$$

Eliminating T from (12) and (16), we obtain

$$\left[\nabla^4 - \nabla^2\left[\frac{C^*}{\bar{K}^*}\left\{(1+\varepsilon)\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right\} + \frac{1}{a^2}\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t} + \frac{C^*}{\bar{K}^*}\frac{1}{a^2}\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial^3}{\partial t^3}\right]q = 0. \tag{19}$$

Similarly, Eqs. (14) and (15) by eliminating $\phi_2[= (-\vec{\phi})_y]$ yield

$$\left[\nabla^4 - \nabla^2\left\{\left(\frac{1}{b^2} + \frac{1}{c_4^2}\right)\frac{\partial^2}{\partial t2} - \frac{\omega_0^2}{c_4^2}\left(\frac{c_3^2}{b^2} - 2\right)\right\} + \frac{1}{b^2 c_4^2}\left(2\omega_0^2 + \frac{\partial^2}{\partial t^2}\right)\right]\psi = 0, \tag{20}$$

where

$$\bar{K}^* = \frac{K^*}{\rho},\quad a^2 = c_1^2 + c_3^2,\quad b^2 = c_2^2 + c_3^2,\quad \psi = (-\vec{U})_y,\quad \varepsilon = \frac{\bar{\nu}^2 T_0}{C^* a^2}$$

Assuming the motion to be harmonic, it follows that

$$(q, T, \psi, \phi_2) = (\bar{q}, \bar{T}, \bar{\psi}, \bar{\phi}_2)e^{i\omega t}. \tag{21}$$

Substituting these expressions in Eqs. (19) and (20), there results

$$(\nabla^4 + A\omega^2\nabla^2 + B\omega^4)\bar{q} = 0, \tag{22}$$

$$(\nabla^4 + C\omega^2\nabla^2 + D\omega^4)\bar{\psi} = 0, \tag{23}$$

where

$$A = \frac{1}{a^2} - \frac{iC^*}{\omega\bar{K}^*}[(1 + \tau_0 i\omega)(1+\varepsilon)],\quad B = -\left(\frac{iC^*}{\omega\bar{K}^* a^2}(1 + \tau_0 i\omega)\right),$$
$$C = \frac{1}{b^2} + \frac{1}{c_4^2} + \frac{\omega_0^2}{c_4^2\omega^2}\left(\frac{c_3^2}{b^2} - 2\right),\quad D = \left(1 - \frac{2\omega_0^2}{\omega^2}\right)\frac{1}{b^2 c_4^2}.$$