Automated quadrilateral mesh generation for digital image structures

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Abstract With the development of advanced imaging technology, digital images are widely used. This paper proposes an automatic quadrilateral mesh generation algorithm for multi-colour imaged structures. It takes an original arbitrary digital image as an input for automatic quadrilateral mesh generation, this includes removing the noise, extracting and smoothing the boundary geometries between different colours, and automatic all-quad mesh generation with the above boundaries as constraints. An application example is provided to demonstrate the usefulness and effectiveness of the proposed approach. [doi:10.1063/2.1106101]

Keywords digital image, microstructure, boundary extracting and smoothing, noise removal, quadrilateral mesh

Digital images are nowadays widely applied to describe the complicated structures with multiple colours in such as mining, medicine and material sciences. A digital image is made up of a rectangular array (or matrix) of equal-sized picture elements. Such elements are usually referred to as "pixels". If a digital photo is repeatedly enlarged without smoothing as photo programs often do, the pixels are seen as squares of constant colour. Therefore, it is possible to describe complex structures in the format of images but such kind of digital models are very difficult and even impossible to be used in the conventional numerical analysis. For example, using finite element method and finite volume method we require high quality mesh/grid as an input. Unstructured quadrilateral mesh generation and refinement have attracted many researchers' interest due to their wide applications in two dimensional (2D) finite element simulations. Image-based finite element mesh construction is the subject of numerous ongoing research.[1–4] How to transform such digital images into high quality unstructured quadrilateral mesh with constraints of multi-colour/-material boundaries is extremely challenging and thus it will be the research focus of this paper.

Take an original arbitrary digital image as an input, the following algorithms are proposed and implemented for automatic quadrilateral mesh generation through four steps:

(1) Noise removal: the noise removal processes with focusing on abstracting characters from images are proposed. Both the number of pixels and characteristic length of a cluster are utilized as criteria parameters, this will prevent from erasing some long and thin features, which are critical to represent multi-colour boundary geometries.

(2) Boundary extraction: in an image, both the border of image and interfaces distinguishing different colours are considered as boundaries which are critical to be defined for numerical analysis. Samples generated beside pixels are used to build edges along the boundaries between different colours. Each pixel has four cardinal directions as well as four intercardinal directions (Fig. 1). The maximum number of sample candidates surrounding a pixel is eight: four at the middle point of each side (i.e. up-, down-, left- and right-side) ($C_i : i = 1, 2, 3, 4$) and four at the corresponding corners ($I_i : i = 1, 2, 3, 4$) (Fig. 1). Take four pixels as a basic unit, and the colour of each pixel is represented by A, B, C or D, respectively. Figure 2 shows the basic rules proposed to generate initial edges (as marked by thick black segments/edges in Fig. 2) for defining the boundaries between different colours at the six different cases. If a border boundary is included, it is treated as a different colour (e.g. C in Fig. 2(c), A or B in Fig. 2(d)). Then each of boundary edges links two different colours or one colour with a border boundary (Figs. 2(a)-2(e)).

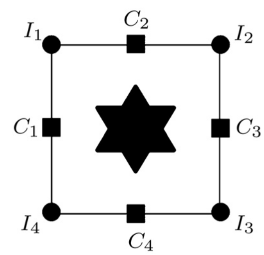

Fig. 1. Candidates of samples and directions of a pixel: square respects cardinal direction and bullet respects intercardinal direction.

(3) Boundary smoothing: due to the nature of digital images, extracted boundary geometries are usually jagged and affect the quality of generated quadrilateral mesh, thus a smoothing process is required. With conventional smoothing methods such as the linear least squares method, the jagged features may be smoothed, but the lengths of segments/edges on the boundary line

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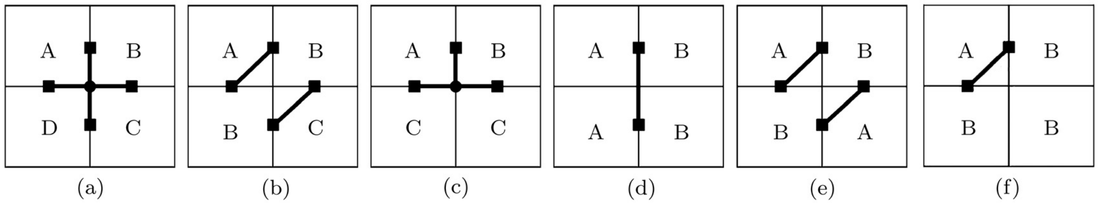

Fig. 2. Six basic cases of sample generation and boundary extraction for pixels with different colours: (a) four; (b) and (c) three; (d), (e) and (f) two.

are normally with significant differences due to the arbitrary features of original image structures. Therefore, a phase-based boundary smoothing method is employed to smooth jagged boundaries and discretise boundaries into edges with ideal sizes. To achieve such a jagged geometry and size smoothing process, samples are treated as observations to fit a quadratic function on a boundary line (i.e. a phase of boundaries). Through the linear least squares method, a boundary line is described by a quadratic function $f(x) = a + bx + cx^2$. Supposing sample S_{i-1}, S_i and S_{i+1} are sequent on this line, we find that the coordinates of node S_{i-1} and S_{i+1} are (x_{i-1}, y_{i-1}) and (x_{i+1}, y_{i+1}), respectively. To smooth the ratio of lengths between edge $S_{i-1}S_i$ and S_iS_{i+1}, the ideal location of S_i on $f(x)$ should satisfy $|S_{i-1}S_i| = |S_iS_{i+1}|$. Hence S_i is moved to the intersection between $f(x)$ and the perpendicular bisector of the curve $S_{i-1}S_{i+1}$. And this intersection is obtained by solving the following equations

$$\begin{aligned} &f(x) = a + bx + cx^2, \\ &g(x) = \frac{y_{i-1} + y_{i+1}}{2} - \frac{x_{i+1} - x_{i-1}}{y_{i+1} - y_{i-1}} \cdot \\ &\qquad \left(x - \frac{x_{i-1} + x_{i+1}}{2}\right), \\ &x_{i-1} < x < x_{i+1}, \end{aligned} \tag{1}$$

where $g(x)$ is the perpendicular bisector of the curve $S_{i-1}S_{i+1}$. The locations of those samples are then updated to discretise the boundary line. The above process is repeated until the ratio of lengths between two neighbouring edges is close to 1 or the user's prescribed value.

(4) Quadrilateral mesh generation: the automatic quadrilateral mesh generation is still difficult especially with complicated line constraints inside, thus the indirect automatic meshing method proposed by authors[4] is utilized here to generate all-quad mesh. Both triangular composing and advancing front technology are combined to generate high-quality all-quad meshes automatically with constraints of a complicated digital image structure. The quadrilateral mesh generation algorithm is outlined in the following steps:

Line constraint segments: The extracted and smoothed boundaries as above are taken as the constrained lines within the domain, which are already identified and discretised into a set of segments/edges as above.

Initial triangular mesh generation: A 2D refinement Delaunay triangulation method[5] is utilized to generate a triangular mesh with the line constraint segments; The feature of the initial triangular mesh including the element gradation, adaptive property as well as the mesh density will be roughly kept through the following quadrilateral mesh generation. The generated triangular grid serves as the initial triangular mesh for quadrilateral mesh generation algorithm.

Quadrilateral mesh transformation: In the first place, a Catmull-Clark subdivision[6] is utilized to split each triangle into three quadrilaterals (which transforms the triangular mesh into an all-quad one); In the second place, based on the generated all-quad mesh, an advancing front technology is performed from the line constraints to generate new quadrilateral elements layer by layer.

Optimization of the quadrilateral mesh: By optimizing the topology of the quadrilateral mesh to reduce the number of irregular nodes, and smoothing the generated mesh, all-quad mesh generation is performed alternatively until a high-quality mesh is generated.

A cross-section image of rock (Fig. 3(a), material properties represented by 8 different colours) is utilized here as an application example to illustrate the proposed method. Firstly, the noises are removed from the original data as illustrated in Fig. 3(b); Secondly, boundaries of different colours/materials are extracted, smoothed and discretised into edges (Fig. 3(c)); Thirdly, the smoothed boundaries are treated as constrained lines to generate a corresponding initial triangular mesh (Fig. 3(d)); Finally, based on this triangular mesh, a reasonable quadrilateral mesh with both boundaries and colours of the original image is generated through combined operations of triangular composing, advancing front technology and mesh optimisation as detailed above (Fig. 3(e)).

In this paper, an automatic quadrilateral mesh generation algorithm is successfully developed for multicolour imaged structures. This includes the following components: removing the noise, extracting, smoothing and discretising the boundary geometries between different colours, and the automatic meshing with the above boundaries as constraints. The generated triangular and quadrilateral mesh can accurately represent the original image by containing the same information such as colours and boundaries. An application example of rock image meshing demonstrated the usefulness