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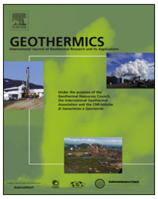

A comparison of analytical and numerical model predictions of shallow soil temperature variation with experimental measurements

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ABSTRACT

In several fields of enquiry such as geothermal energy, geology and agriculture, it is of interest to study the thermal behaviour of shallow soils. For this, several analytical and numerical methodologies have been proposed to analyse the temperature variation of the soil in the short and long term. In this paper, a comparative study of different models (sinusoidal, semi-infinite and finite difference method) is conducted to estimate the shallow soil temperature variation in the short and long term. The models were compared with hourly experimental measured data of soil temperature in Leicester, UK, at depths between 0.75 and 2.75 m. The results show that the sinusoidal model is not appropriate to evaluate the short-term temperature variations, such as hourly or daily fluctuations. Likewise, this model is highly affected by the undisturbed ground temperature and can lead to very high errors. Regarding the semi-infinite model, it is accurate enough to predict the short-term temperature variation. However, it is useless to predict the long-term variation at depths greater than 1 m. The finite difference method (FDM) considering the air temperature as a boundary condition for the soil surface is the most accurate approach for estimating both short and long-term temperature variations while the FDM with heat flux as boundary condition is the least accurate approach due to the uncertainty of the assumed parameters. The ranges of errors for the sinusoidal, semi-infinite and FDM are found to be from 76.09 to 142.13%, 12.11 to 104.88% and 1.82 to 28.14% respectively.

1. Introduction

The thermal behaviour of the shallow soil has been of great interest in different research fields such as agriculture (Wullschleger et al., 1991), geology (Singh et al., 2017), low enthalpy geothermal energy (Signorelli and Kohl, 2004). The last of these has been of particular interest in recent years due to the increasing use of geothermal heat pump systems for heating and cooling applications. In the case of conventional geothermal heat pump systems, the soil is considered to be a stable and homogeneous medium (Lamarche and Beauchamp, 2007). This assumption is valid at depths where the ground temperature is undisturbed, usually at a depth greater than 30 m (Sharqawy et al., 2013). However, for systems that feature shallow boreholes or horizontal ground heat exchangers (sometimes called ‘slinkies’), seasonal variations as well as changes in the soil temperature in the short term (daily or weekly variations) cannot be neglected. Such systems are usually installed at depths no greater than 5 m, where they are highly influenced by environmental conditions such as ambient temperature, solar radiation, rainfall and groundwater flows (Bidarmaghz et al., 2016). Therefore, models that accurately predict the influence of these factors, to estimate the thermal behaviour of the soil at different depths over time, should be considered. For this purpose, physical (analytical or numerical) or empirical models can be used. Physical models consider the mechanisms of heat or heat-moisture transfer between the soil and the environment (Wullschleger et al., 1991) and can be adapted to different locations, soil typologies and environmental conditions. Empirical models are based on correlations from experimental data or time series modelling, are simpler to apply but not always adaptable to different soil typologies or environmental conditions (Droulia et al., 2008). To develop accurate time series models a large data set of input data is required (Shirvani et al., 2015). For this reason, physics models are preferred for shallow geothermal applications as usually large datasets are unavailable.

1.1. Analytical physical models

Several studies using analytical physical models have considered the soil temperature to have a harmonic (sinusoidal) variation over time

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Nomenclature

α	thermal diffusivity [m^2/s]
δ	penetration depth [m]
ε	emissivity
φ	phase angle [rad]
γ	psychometric constant
ρ	density [kg/m^3]
σ	Stephan-Boltzmann constant [kg/m^3]
ω	angular frequency [rad/time]
ΔT	temperature difference [°C]
E	evaporation rate [kg/m^2s]
E_p	evaporation potential rate [kg/m^2s]
Fo	Fourier number
G	ground surface heat flux [W/m^2]
H	convection [W/m^2]
I	solar radiation [W/m^2]
K	von Karman constant
L	latent heat of vaporization [J/kg]
LE	evaporation heat [W/m^2]
LR	long wave radiation [W/m^2]
R_n	net radiation [W/m^2]
P	rainfall [mm/s]
SR	short wave radiation [W/m^2]
T	temperature [°C]
abs	soil absorptivity
c_p	specific heat [J/kg°C]
e_a	actual vapour pressure [kPa]
e_s	saturation vapour pressure [kPa]
k	thermal conductivity [W/mK]
m	node element
p	time element
q''	heat flux [W/ m^2]
r	aerodynamic resistance
t	time [s]
u	wind speed [m/s]
z	depth [m]

Subscripts

a	air
d	daily
gr	ground
in	inlet
m	measurement height
o	initial
s	soil
sky	surroundings
st	stored
out	outlet

(Chow et al., 2011). For instance, one of the most accepted sinusoidal models is that of Kusuda and Achenbach (1965). This approach is most accurate at depths where short-term (hourly and daily) changes in the soil's thermal behaviour can be neglected, which is generally at depths greater than 1 m. Likewise, the semi-infinite solid model (Incropera et al., 2007) can be used to study transient phenomena in solids where the heat diffusion is predominantly one dimensional. This model is accurate to study short-term variations in the soil temperature but only at very shallow depths (no more than a few centimetres).

Several studies have proposed different models to estimate the variation of soil temperature. For example, Charpin et al. (2004) performed an analytical study using a sinusoidal harmonic model of the heat transfer in a concrete block exposed to the environment. In their study, the authors used the soil surface exposed to a heat flux that includes solar radiation and convection, as a boundary condition. They used the harmonic model to define the variation of ambient air temperature. The authors report that, for a one-day period, the concrete block shows variations in its temperature up to a depth of 20 cm (thermal penetration depth). The model did not allow accurate predictions of the hourly temperature variation but was suitable for a first estimation although the results of the model were not experimentally validated. Likewise, Cleall et al. (2015) proposed an analytical model to estimate the soil temperature based on harmonic variations of global solar radiation and ambient temperature. The resolution of the analytical model was compared with a numerical model showing an acceptable match. Although obtaining the analytical solution is complex, the implementation of the final model is simple, and its calculation is fast. This model serves to accurately estimate the thermal behaviour of the soil, although short-term variations cannot be estimated given the harmonic principle of the model. The harmonic model correlated well ($R^2 = 0.96$) with the experimental data at a depth of 1 m however the correlation was poor ($R^2 = 0.63$) at a depth of 0.025 m since short-term variations are not represented well by a harmonic function. In another study, Badache et al. (2016) propose an analytical model, based on the Kusuda and Achenbach model, in which the boundary condition is a surface temperature value determined by an empirical model. This method reduces the complexity in obtaining the analytical solution. The model was validated with monthly experimental data from three different locations at depths between 0.1 and 4 m. The validation shows a good correlation with a root mean square error (RMSE) of 2.5 K in the worst case at 1 m depth. However, a limitation of this model is its inability to represent the shallow soil behaviour in the short-term (hourly or daily variations). Due to the complexity of the actual interaction between the soil and the environment, analytical models only deal with the heat transfer phenomenon and neglect the moisture transport process.

1.2. Numerical physical models

Numerical models are known to be more accurate and robust for the study of different soil typologies and boundary conditions (BC). However, numerical methods are more complex to implement and take longer to solve (Droulia et al., 2008). Usually, constant surface temperature or constant heat flux (Cleall et al., 2015) are applied as boundary conditions at the soil surface level. For either of these, it is necessary to determine the soil surface temperature which is normally a parameter that cannot be easily determined since conventional meteorological stations do not measure it (Holmes et al., 2012). To deal with this problem, in some cases, the soil surface temperature can be approximated as the air temperature (Charpin et al., 2004). As numerical models are able to represent periodic boundary conditions as well as a more realistic approach to the interaction between soil and environment (including heat and moisture transfer), numerical studies have demonstrated a greater accuracy than analytical models for the study of the thermal behaviour of soil. In fact, Yilmaz et al. (2009) concluded in their study that analytical models are generally unrealistic for soil temperature prediction. They compared a harmonic model with a numerical model using the finite difference method (FDM) with heat flux at the surface as a boundary condition. This numerical model allowed the short-term fluctuations to be estimated, however the model was not validated with experimental data. Wullschleger et al. (1991) developed a computational tool that numerically predicts the soil temperature variation. They included heat and moisture transfer phenomena in the model, which required the precipitation or water irrigation in the soil as input data. The model is able to predict hourly temperature variations on a daily basis. However, it lacks experimental