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Operator-based linearization for efficient modeling of geothermal processes

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ABSTRACT

Numerical simulation is one of the most important tools required for financial and operational management of geothermal reservoirs. The modern geothermal industry is challenged to run large ensembles of numerical models for uncertainty analysis, causing simulation performance to become a critical issue. Geothermal reservoir modeling requires the solution of governing equations describing the conservation of mass and energy. The robust, accurate and computationally efficient implementation of this solution suggests an implicit time-approximation scheme, which introduces nonlinearity into the system of equations to be solved. The most commonly used approach to solving the system of nonlinear equations is based on Newton's method and involves linearization with respect to nonlinear unknowns. This stage is the most complicated for implementation and usually becomes the source of various errors. A new linearization approach – operator-based linearization – was recently proposed for non-isothermal flow and transport. The governing equations, discretized in space and time, were transformed to the operator form where each term of the equation was specified as the product of two operators. The first operator comprises physical properties of rock and fluids, such as density or viscosity, which depend only on the current state of a grid block, fully defined by the values of nonlinear unknowns. The second operator includes all terms that were not included in the first operators, and depends on both the state and spatial position of a control volume. Next, the first type of operators was parametrized over the physical space of a simulation problem. The representation of highly nonlinear physics was achieved by using multi-linear interpolation, which replaces the continuous representation of parametrized operators. The linearization of the second type of operators was applied in the conventional manner. In this work, we investigated the applicability of this approach to the geothermal processes, specifically for low-enthalpy and high-enthalpy geothermal doublet models with hydrocarbon co-production. The performance and robustness of the new method were tested against the conventional approach on a geothermal reservoir of practical interest. This approach shows significant improvement of geothermal simulation performance, while errors, introduced by coarsening in physics, remain under control. The simplicity of implementation on emerging computational architectures and nonlinearity reduction provide advanced opportunities for uncertainty quantification and risk analysis of geothermal projects.

1. Introduction

The modern development of geothermal resources requires high-performance numerical reservoir simulations. Numerical models are used to predict and compare the performance of different reservoir-development schemes, defined by, e.g., the lifetime of a geothermal doublet in the case of low-enthalpy reservoirs or the recovery of a geothermal reservoir. Complex and fine-scale models enhance prediction accuracy but demand more computational resources. Large ensembles of reservoir models are run to perform sensitivity analysis and reduce uncertainties in parameters estimation. The quality of these processes depends on the number of models in ensembles, which is limited by the computational time required for a single model. Therefore, efficient reservoir simulation performance is essential for geothermal industry: any noticeable improvement could positively affect production workflow.

Numerical geothermal reservoir simulation requires discretization of governing Partial Differential equations (PDE), which describe mass and energy transport in a reservoir. The combination of Finite Volume discretization in space and Fully Implicit Method (FIM) approximation in time provides a robust, accurate and efficient modeling approach (Aziz and Settari, 1979). However, an implicit nature of the time approximation increases the nonlinearity of the governing equations. For those geothermal models that use both gas and liquid phases, complex multiphase behavior and the assumption of thermodynamic equilibrium further amplify nonlinearity.

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In reservoir simulation, Newton-Raphson's method has become a standard solution for solving the nonlinear system of equations by linearizing it. The linearization is one of the most important and challenging components of a reservoir simulation framework. This step requires the determination of the derivatives of all residual equations with respect to independent variables. The particular set of independent variables (i.e., nonlinear unknowns) is defined by a nonlinear formulation. The flexibility of linearization reflects the simplicity of changing the nonlinear formulation in an existing simulation framework. Based on the formulation, all properties and their derivatives need to be determined and assembled into the matrix of partial derivatives (Jacobian). The linearization stage defines the accuracy and robustness of nonlinear solution, dictates the data layout of a linear system and therefore has a great impact on the reservoir simulation performance.

There are three most commonly used linearization approaches. The first one, numerical derivatives, provides flexibility in the implementation (see Pruess, 2006 for example), but usually lacks robustness (Vanden and Orkwis, 1996) and may lead to a stalled behavior in some cases (O'Sullivan et al., 2014). The second one, straightforward analytical derivations, requires fixing the nonlinear formulation and physical models used in a computational framework and that often limits its flexibility (e.g Geoquest, 2011). Finally, Automatic Differentiation (AD) technique provides both flexibility and robustness to the development of simulators. In reservoir simulation, an Automatic Differentiation General Purpose Research Simulator (ADGPRS) was developed for simulation of generic thermal-compositional problems (Voskov and Tchelepi, 2012; Zaydullin et al., 2014). ADGPRS is a unified reservoir simulation framework providing an extensive set of nonlinear formulations (Voskov, 2011; Zaydullin et al., 2013), including different formulations for a geothermal model (Wong et al., 2015; Wong et al., 2016) and a generic treatment of complex phase behavior (Iranshahr et al., 2010; Iranshahr et al., 2013). Unfortunately, the AD technique by design inherits computational overhead and therefore decreases reservoir simulation performance. Thereby, there is a clear demand for robust, flexible and computationally efficient linearization approach.

A new operator-based linearization (OBL) approach was recently introduced by Voskov (2017). It suggests a different way of linearization, making use of the discrete representation of the physics. In the governing PDE, discretized in time and space, the terms which depend only on state variables are approximated by piece-wise multilinear operators. For a given problem, the current physical state fully defines operators. Therefore, each operator can be parametrized over the multidimensional space of nonlinear unknowns for a given distribution of supporting points, resulting a set of tables. In the course of a simulation, the values of the operators, as well as partial derivatives with respect to nonlinear unknowns, are obtained from the tables using multilinear interpolation. Being as flexible as numerical derivatives, OBL is more consistent and reliable, since the operators are piecewise multilinear functions with derivatives related to the interpolation coefficients. Thus the proposed approach always provides an accurate Jacobian, avoiding stalled behavior and instabilities, which may occur due to the application of numerical derivatives (e.g. challenges in THOUGH2 simulation described by Vanden and Orkwis, 1996; Noy et al., 2012). In addition, it reduces the level of nonlinearity of the problem to be solved, providing a significant improvement in nonlinear solver performance.

In the original approach of Voskov (2017), OBL was described and applied to isothermal hydrocarbon systems. The version of the OBL approach with adaptive parametrization of physical operators was implemented in the ADGPRS framework and tested for petroleum applications in Khait and Voskov (2017a). A special version of a stand-alone isothermal prototype with a limited number of components was implemented in both CPU and GPU architectures in Khait and Voskov (2017b). Here, we farther extend the new prototype framework to thermal systems and investigate an applicability of the OBL method to simulation of geothermal processes, including low-enthalpy and high-enthalpy systems with hydrocarbon co-production. First, we briefly describe the modeling approach. Next, we apply OBL to a realistic geothermal reservoir and compare the performance and solution accuracy against the conventional approach, implemented in ADGPRS. The new simulation prototype demonstrates better performance of simulations with discrete representation of physics where the resolution of interpolation tables controls an approximation error. The proposed simulation framework becomes the essential tool for uncertainty quantification and risk analysis where the performance of forward simulation is the major limiting factor.

2. Modeling approach

Here we describe the state-of-the-art modeling approach of a non-isothermal multiphase multicomponent flow, which was used as a reference solution, followed by the description of operator-based linearization (OBL).

2.1. Conventional modeling approach for thermal compositional problem

This section contains the description of a thermal multiphase compositional problem with n_p phases and n_c components. This problem can be described by n_c equations of mass conservation and one energy equation:

$$\frac{\partial}{\partial t}\left(\phi \sum_{j=1}^{n_p} x_{cj}\rho_j s_j\right) - \operatorname{div}\sum_{j=1}^{n_p} x_{cj}\rho_j\left(K\frac{k_{rj}}{\mu_j}(\nabla p_j - \gamma_j \nabla D)\right) \quad c = 1,\ldots,n_c + \sum_{j=1}^{n_p} x_{cj}\rho_j \tilde{q}_j = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\phi\sum_{j=1}^{n_p}\rho_j s_j U_j + (1-\phi)U_r\right) - \operatorname{div}\sum_{j=1}^{n_p} h_j\rho_j\left(K\frac{k_{rj}}{\mu_j}(\nabla p_j - \gamma_j\nabla D)\right) + \operatorname{div}(\kappa\nabla T) + \sum_{j=1}^{n_p} h_j\rho_j\tilde{q}_j = 0, \tag{2}$$

where:

- ϕ – porosity,
- x_{cj} – the mole fraction of component c in phase j,
- s_j – phase saturations,
- ρ_j – phase molar density,
- K – permeability tensor,
- k_{rj} – relative permeability,
- μ_j – phase viscosity,
- p_j – pressure in phase j,
- γ_j – gravity vector,
- D – depth (backward oriented).
- U_j – phase internal energy,
- U_r – rock internal energy,
- h_j – phase enthalpy,
- κ – thermal conduction.

Next, a finite-volume discretization on a general unstructured mesh and backward Euler approximation in time are applied:

$$V\left[\left(\phi\sum_{j=1}^{n_p} x_{cj}\rho_j s_j\right)^{n+1} - \left(\phi\sum_{j=1}^{n_p} x_{cj}\rho_j s_j\right)^{n}\right] \quad c = 1,\ldots,n_c - \Delta t\sum_{l}\left(\sum_{j=1}^{n_p} x_{cj}^{l}\rho_j^{l}\Gamma_j^{l}\Delta\psi^{l}\right) + \Delta t\sum_{j=1}^{n_p}\rho_p x_{cj} q_j = 0, \tag{3}$$