Dual solutions in mixed convection with variable physical properties

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Abstract Assisting and opposing flows in a mixed convection boundary layer flow over an isothermal vertical plate are studied for the case of variable physical properties and uniform free stream. Fluid viscosity and thermal conductivity are assumed to be linear functions of temperature. Using local similarity the flow and heat transfer quantities are found to be functions of four parameters, i.e. Richardson number, Prandtl number, a viscosity variation parameter and a thermal conductivity variation parameter. Numerical solutions are obtained by two methods, a shooting technique and Nachtsheim-Swigert technique, for selected values of parameters appropriate for the fluids considered and specific temperatures of the plate and ambient fluid. For assisting flows, there exist solutions for all values of Richardson number while for opposing flows solutions exist only for a finite set of its values and, in addition, there also exist dual solutions. Important flow and heat transfer quantities of practical interest are determined and the influence of different parameters is discussed. [doi:10.1063/2.1102206]

Keywords mixed convection, assisting flow, opposing flow, dual solutions

Mixed convection flows arise in many transport processes in engineering devices and in nature. As a result, extensive studies of mixed convection flows were made by researchers over the past five decades. Lin and Chen[1] gave a detailed account of certain mixed convection studies made during 1958 and 1986. In all the works quoted in Ref. 1 results are presented for positive Richardson number Ri and for different Prandtl number Pr. In Acrivos,[2] Lin and Chen,[1] the conventional mixed convection parameter, Ri is replaced by other parameters, but, results are presented only for positive values of appropriate parameters. Llyod and Sparrow[3] presented solutions to the aiding mixed convection flow at an isothermal vertical pl- ate by using local non-similarity method. Gryzagoridis[4] made an experimental study of mixed convection at an isothermal vertical plate and presented a comparison between theoretical and experimental values of temperature, velocity and heat transfer rates. Merkin and Pop[5] considered aiding and opposing flows in mixed convection and presented solutions for both positive and negative values of Richardson number. They also obtained dual solutions in the opposing flow case and have shown qualitative differences in the flow structure for aiding and opposing flows. In all the works quoted above, fluid properties are taken to be constant. Usually, fluid properties are taken to be constant in theoretical analysis while in many situations they are temperature dependent. In view of this, a number of works were carried out to study the effect of variable fluid properties (VFP) on different convection flows. Kafoussias et al.[6] and Chin et al.[7] treated viscosity as a function of temperature and presented dual solutions with Prandtl number taking certain representative values. Merkin[8] and Ishak et al.[9–11] presented dual solutions for the case of constant fluid properties (CFP) in different convection flows in a porous medium. To the best knowledge of the authors, dual solutions have not yet been discussed for mixed convection boundary layer flows with variable viscosity and variable thermal conductivity in assisting and opposing flows (in free flow).

In the present analysis, mixed convection boundary layer flow at an isothermal vertical plate with uniform free stream, variable viscosity and variable thermal conductivity (in mercury, air and water) over specific temperature ranges is discussed. Local non-similarity method as proposed by Llyod and Sparrow[3] is utilized to analyze the problem.

Owing to the variation of properties with temperature, two parameters, i.e. a viscosity variation parameter γ_μ and a thermal conductivity variation parameter γ_k, arise besides Richardson number ξ and Prandtl number Pr. Also, depending on the direction of external free stream and temperatures of the plate and ambient fluid, there exist four cases, of which, two correspond to assisting flow and the other two correspond to opposing flow. In the present problem, only two cases (1) assisting flow with hot plate and (2) opposing flow with cold plate are considered.

Consider a semi-infinite vertical stationary plate at temperature T_w in a viscous incompressible fluid of ambient temperature T_∞ flowing with a uniform velocity U_∞. X-axis is taken vertically upwards along the plate and Y-axis perpendicular to it. Under the Boussinesq approximation the steady state mixed convection boundary layer equations are well known. Following Carey and Mollendorf,[12] fluid viscosity μ is assumed to vary with temperature as $\mu = \mu_f s_\mu(T)$ and, in a similar way, thermal conductivity k is assumed to vary with temperature as $k = k_f s_k(T)$. Appropriate boundary

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conditions on the velocity and temperature fields are

$$y = 0 : u = 0,\ v = 0,\ T = T_w,$$
$$y \to \infty : u \to U_\infty,\ T \to T_\infty. \quad (1)$$

Using local similarity method the co-ordinates are transformed from (x, y) system to (ξ, η) system where $\eta = \eta(x, y)$ and $\xi = \xi(x)$. Here η, a pseudo-similarity variable and ξ, a mixed convection buoyancy parameter, also known as Richardson number, are defined as

$$\eta = \frac{y}{x}\sqrt{\frac{Pe_x}{2}},\ \xi = Ri = \frac{Gr_x}{Re_x^2} \approx x. \quad (2)$$

By introducing non-dimensional stream function $f(\xi, \eta)$ and non-dimensional temperature $\phi(\xi, \eta)$ through relations

$$2\alpha\sqrt{Pe_x/2}f(\xi, \eta) = \psi(x, y),$$

$$\phi(\xi, \eta) = (T - T_\infty)/(T_w - T_\infty)$$

(where $\psi(x, y)$ is the conventional stream function) into the governing equations, when viscous dissipation and volumetric heat generation are neglected, the following transformed equations are obtained.

$$S_\mu \frac{\partial^3 f}{\partial \eta^3} + \gamma_\mu \frac{\partial \phi}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2} + \frac{1}{Pr}\left(2\xi\phi + f\frac{\partial^2 f}{\partial \eta^2}\right)$$
$$= \frac{2x}{Pr}\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi \partial \eta} - \frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right), \quad (3)$$

$$S_k \frac{\partial^2 \phi}{\partial \eta^2} + \gamma_k \left(\frac{\partial \phi}{\partial \eta}\right)^2 + f\frac{\partial \phi}{\partial \eta}$$
$$= 2x\left(\frac{\partial f}{\partial \eta}\frac{\partial \phi}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial \phi}{\partial \eta}\right). \quad (4)$$

Here $S_\mu(\phi) = 1 + \gamma_\mu(\phi - 1/2)$, where γ_μ is a viscosity variation parameter and $S_k(\phi) = 1 + \gamma_k(\phi - 1/2)$ and γ_k is a thermal conductivity variation parameter. Film temperature corresponds to $\phi = 1/2$ so that $S_\mu(1/2) = S_k(1/2) = 1$.

The boundary conditions become

$$f(\xi, 0) = 0,\ \frac{\partial f}{\partial \eta}(\xi, 0) = 0,\ \phi(\xi, 0) = 1.$$
$$\frac{\partial f}{\partial \eta}(\xi, \infty) = 1,\ \phi(\xi, \infty) = 0. \quad (5)$$

In accordance with the principle of local similarity, the ξ derivatives are deleted from the transformed Eqs. (3) and (4). At any stream wise position along the plate, the quantity ξ may be regarded as an assignable constant parameter. As a consequence, Eq. (3) and (4) may be treated as a system of ordinary differential equations at each stream wise location of interest. Now the reduced system of ordinary differential equations are

$$Pr(S_\mu f''' + \gamma_\mu f''\phi') + 2\xi\phi + ff'' = 0, \quad (6)$$

$$S_k\phi'' + \gamma_k\phi'^2 + f\phi' = 0, \quad (7)$$

where a dash denotes derivative with respect to η.

Prandtl number is a fluid property and it varies considerably with temperature. So fluids in certain ranges of temperatures are considered and the solutions are obtained for four values of Prandtl number $Pr = 0.01, 0.71, 4.87\ \&\ 7.0$ that correspond to Mercury, air, saturated water and water at atmospheric pressure at appropriate film temperatures. Richardson number ξ provides a measure of the influence of free convection in comparison with that of forced convection on the fluid flow. Solutions are obtained for a few positive values of ξ that correspond to assisting flows, for zero value of ξ that corresponds to forced convection and selected negative values of ξ that correspond to opposing flows. In the opposing flow case, it is also obtained that for certain ranges of values of ξ either a unique solution or dual solutions or no solution exists. Depending on the fluids and on the temperatures T_w and T_∞, the parameters γ_μ and γ_k can assume both positive and negative values. The limiting values for both the parameters are -2 and $+2$. Zero values of γ_μ, γ_k represent CFP (constant fluid properties) case whereas non-zero values of γ_μ, γ_k represent VFP (variable fluid properties) case. In the opposing flow case solutions are obtained for γ_μ =1.0,γ_k=- 0.1 for $Pr = 7.0$; $\gamma_\mu = 1.1634$, $\gamma_k = -0.1118$ for $Pr = 4.87$; $\gamma_\mu = -0.1$, $\gamma_k = -1.0$ for $Pr = 0.71$; $\gamma_\mu = 0.4$, $\gamma_k = -0.3$ for $Pr = 0.01$. Appropriate values are taken for γ_μ, γ_k in the assisting flow case. Some solutions are also obtained for zero values of γ_μ, γ_k for assessing the effect of variable fluid properties on the flow and heat transfer and also for comparison with earlier works.

Equations (6) and (7) subjecting to boundary conditions (5) are solved by Runge-Kutta-Gill method coupled with a shooting technique. The equations are also solved by Nachtsheim-Swigert technique. Excellent agreement is observed between the solutions obtained by the two techniques. In the opposing flow case, for negative values of ξ, two flow and heat transfer states are observed to exist and they are referred to as dual solutions. One of the solutions that correspond to a relatively larger value of $f''(0)$ is referred to as the upper solution and the other one as the lower solution.

Certain representative plots of skin friction and wall heat transfer rate of the opposing and assisting flow cases are presented in Figs. (1)–(4). It is noticed from Fig. 1 and also from numerical results that, in the opposing flow case, a single solution exists for certain negative values of ξ, dual solution exist for a range of values of ξ and no solution exists beyond a critical value of ξ. Absolute critical values of ξ increase with increasing values of Prandtl number Pr.

In the opposing flow case, for all fluids considered (i.e. for all Prandtl numbers), skin friction $f''(0)$ assumes both positive and negative values. In this case, for negative values of ξ, the skin friction diminishes with diminishing values of ξ up to a certain stage, then be-