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A simple model of gravitationally-driven water flow in a semicircular aquifer to estimate geothermal power potential: Examples from Arizona and Colorado

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ABSTRACT

An analytical model has been developed from which the basic thermal parameters of the flow through non-magmatic hot-spring systems can be calculated from data that can be obtained at their discharge: water outflow temperature, mean surface temperature, and water geochemistry. From this flow, the maximum thermal power potential of a system can be calculated. The model has been applied to Clifton Hot Springs in eastern Arizona, USA, and the potential power generation of this system was estimated to be within the range of 0.8–3 MW. The model has also been successfully applied to three hot spring systems in Colorado with different characteristics. The simple model is not thought to represent the details of flow in the aquifers, but provides a useful approximation of the temperatures and thermal power potential in the systems.

1. Introduction

Hot springs, geysers, fumaroles, and other surface thermal features are the surface manifestations of subsurface geothermal systems. These subsurface systems may be driven by 1) localized magmatic heating of the upper crust and/or 2) heating of groundwater in permeable upper crustal rocks by regional heat flow and the resulting regional geothermal gradient, which may be locally increased by low thermal conductivity rocks where the circulation occurs. If the geothermal gradient is sufficiently high, and the permeability sufficiently high, groundwater circulation may be driven by thermal buoyancy. More commonly, heat is transferred by groundwater movements driven primarily by piezometric gradients (caused by changes in the elevation of the water table) with thermal buoyancy forces a secondary factor.

There are two end members for models of geothermal systems. At one extreme, models seek to be as realistic as possible including everything that is known about the geology of the systems, the physics and the chemistry of the heat transfer and fluid behavior in the system. These models are typically complex numerical computer simulations. At the other extreme are analytical simplifications of the plumbing, flow, and heat transfer within the systems, often ignoring the chemistry of the systems. These models may use dimensionless parameterization of the systems so that they may be applied to a wide range of scales without repeated computations. They are not expected to be an accurate simulation of any particular geothermal system, but they provide useful order-of-magnitude estimates of the thermal parameters of classes of geothermal systems.

In this contribution, an analytical hot-spring model is developed that is a simplification of water flow in a vertical fracture system, and the model is applied to data from a specific hot spring system in eastern Arizona and more general examples from Colorado. The model assumes that the path of water flow driven by piezometric (gravity-induced) gradients in near-vertical fractures may be approximated by a simple geometrically-shaped path that has a uniform cross-section along the path. The basic solution for the shape chosen, a semi-circle, has already been solved, but requires information about the system, such as maximum depth of water flow, that is not readily available. However, the solution is developed further here to show that with only information about the system that can be collected at the system's discharge, including water outflow temperature, mean surface temperature, and major element chemistry of the outflow waters, the basic thermal parameters, including the thermal power potential of the system, can be calculated.

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Nomenclature

Pe	Péclet number (dimensionless)
R	Mean radius of curvature of semicircular aquifer (m or km)
r	Radius of the cross-section of the semicircular aquifer (m or km)
T	Temperature (°C)
T_m	Maximum water temperature in aquifer (°C)
T_O	Water outflow (discharge) temperature (°C)
T_R	Undisturbed wall rock temperature at the maximum depth penetrated by the aquifer (R) (°C)
T_s	Temperature at surface (°C)
$\bar{u}$	Mean laminar flow velocity in the aquifer (m/s or km Ma^{-1})
$\overline{V}$	Aquifer discharge rate (l/s—liters per second)
z	Depth (m or km)

Greek symbols

β	Geothermal gradient (mK m^{-1} = °C km^{-1})
ζ	Shorthand notation for [(48R)/(11rPe)]
θ	Dimensionless temperature
θ_e	Dimensionless water exit (discharge) temperature
θ_{max}	Maximum dimensionless water temperature in aquifer
κ	Rock thermal diffusivity (mm^2 s^{-1} or km^2 Ma^{-1})
π	Mathematical π
ϕ	Angular distance along the semicircular aquifer (radians)
ψ	Dimensionless flow rate

2. Semicircular aquifer model

Groundwater flow is assumed to be driven by piezometric (gravitationally-derived) pressure gradients usually associated with topography and recharge. Recharge to the groundwater system is typically greatest in areas of high elevation resulting in a high water table and high piezometric pressure. Water flows to areas of lower elevations where the water table is lower and piezometric pressures are lower. In a homogeneous earth, the resulting flow would be greatest at the water table, diminishing in magnitude with depth. In a heterogeneous earth, flow is concentrated through aquifers and permeable fracture zones and is diverted by aquitards.

Hot springs are formed where groundwater flows to depth before returning to the surface. Such flow may be part of a large basin system, for example, in the hot springs between the basins of the Rio Grande rift (Morgan et al., 1981), or in more restricted vertical fracture systems, as with the Clifton Hot Springs. In both types of flow systems a significant component of deep flow in the system must be present, and the velocity of this flow must be balanced: too high of a velocity results in cooling of the rocks at depth and too slow of a velocity results in cooling of the groundwater as it returns to the surface. Simple models may be used to estimate the effective ranges of flow velocities to produce hot springs in flow systems with different geometries (e.g., Morgan et al., 1981). Vieira et al. (2014) proposed a useful, simple model that can be adapted to investigate heat exchange in geologic structures with non-circular geometry. However, this model requires sets of temperature gradients and Péclet numbers, rather than data that may be collected at the spring discharge, as with the semi-circular aquifer model.

The simplest geometry that may be used to model flow in a vertical fracture assumes that the permeable zone, or aquifer, is circular in cross section and semi-circular in its path, as shown in Fig. 1 (Turcotte and Schubert, 2002). These assumptions are clearly simplistic. However, they allow testing of the relative importance of basic parameters such as geothermal gradient, aquifer surface area, and flow rate.

Assuming a vertical thermal gradient in the earth, β, water is heated as it descends through the aquifer and cools as it ascends to the surface, depending on the water flow rate. This heating and cooling may be demonstrated for the general case by formulating the problem in dimensionless numbers and assuming laminar flow, as shown in Fig. 2 (see Appendix A for the formulae to calculate these temperatures). The maximum temperature in the aquifer and the exit temperature from the aquifer (hot spring temperature) depend on the flow rate, as shown in Fig. 3 (formulae for calculation of these temperatures are also given in the Appendix A).

Figs. 2 and 3 are plotted using dimensionless temperature θ on the ordinate (y) axes and this is defined as:

$$\theta = \frac{(T - T_s)}{(T_R - T_s)}, \tag{1}$$

where T is temperature, T_s is surface temperature, and T_R is the undisturbed wall-rock temperature at the maximum depth penetrated by the aquifer (controlled by the geothermal gradient). In Fig. 3 the dimensionless temperature on the ordinate axis is the water exit (hot spring) temperature, θ_e.

Dimensionless flow rate is defined by (r Pe)/R, where Pe is the Péclet number defined as:

$$Pe = \frac{\bar{u}r}{\kappa} \tag{2}$$

where $\bar{u}$ is mean laminar flow velocity in the aquifer, R is the radius of curvature of the semicircular aquifer, κ is the thermal diffusivity, and the aquifer has a circular cross section of radius r.

For a typical hot spring, parameters that are easily measured or available are outflow temperature, T_O, and surface temperature (mean annual surface temperature, T_s). Water geochemistry may be used to estimate the reservoir temperature of the aquifer feeding the hot spring, T_m, (Fournier and Rowe, 1966; Fournier and Truesdell, 1973; Fournier, 1977), and as fracture systems are unlikely to have large-volume reservoirs, the "reservoir temperature" estimated by groundwater geochemistry is likely to be close to the maximum temperature reached by the groundwater flowing through the fracture system. As shown in Fig. 2, this maximum temperature decreases with increasing flow rate, (r Pe)/R, and the position at which the groundwater reaches its maximum temperature moves toward the flow outlet as flow rate increases. We can determine the maximum dimensionless temperature for each dimensionless flow rate (see Appendix A) shown in Fig. 2 and make a third dimensionless parameter plot of the ratio of dimensionless exit temperature θ_e (outflow temperature minus mean annual surface temperature defined as in Eq. (1)) to maximum dimensionless temperature θ_{max} versus dimensionless flow rate. Taking the ratio of the dimensionless exit temperature to the maximum temperature removes the need to know the maximum wall-rock temperature in calculation of the dimensionless temperatures as it cancels when the ratio is taken (Eq. (3)). This plot is shown in Fig. 4.

Ratio of dimensionless exit temperature to dimensionless maximum temperature =

$$\frac{\theta_e}{\theta_{max}} = \frac{(T_O - T_s)}{(T_R - T_s)} \Big/ \frac{(T_m - T_s)}{(T_R - T_s)} = \frac{(T_O - T_s)}{(T_m - T_s)} \tag{3}$$

How may these dimensionless plots be applied to the real world? For most hot springs, the mean annual surface temperature is already known, or it can be measured. The outflow temperature can be measured. The maximum reservoir temperature may be estimated from water geochemistry. Using these data, the ratio of dimensionless exit temperature to dimensionless maximum temperature (Eq. 3) may be used to estimate the dimensionless flow rate from Fig. 4. Using this estimate of dimensionless flow rate, the dimensionless maximum temperature in the aquifer θ_{max} and exit temperature θ_e from the system may be determined from either