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Modelling viscous fingering during reinjection in geothermal reservoirs

Alastair Mcdowell, Sadiq J. Zarrouk*, Richard Clarke

Department of Engineering Science, The University of Auckland, Private Bag 92019, Auckland, New Zealand

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ABSTRACT

Viscous fingering is a hydrodynamic instability that can occur when two fluids of differing viscosity come into contact, and leads to complex intrusions (or fingers) of one fluid into the other. We are interested here in this instability within the context of reinjection into a geothermal reservoir, where the viscosity of the injected fluid differs from that in the reservoir due to its higher temperature. We first perform a classical linear stability analysis, which considers the early time growth of temperature perturbations. Here the governing equations can be simplified through linearisation, and this linear stability analysis shows which wave number, and hence fingering displacement shape, will initially be most unstable. Results illustrate that there are two key parameters that define the stability: the Peclet number (which is proportional to injection rate), and the log-mobility ratio (which is related to injection enthalpy). These critical parameters provide limits on injection temperature and flow rate that will be stable against the fingering instability for a reservoir of given temperature, and we see that the values of these suggest that viscous fingering will always likely occur in practice. We also use the TOUGH2 numerical simulator (Pruess et al., 1999) to solve the full (i.e. non-linearised) governing equations, and confirm that viscous fingering is indeed occurring at such low injection rates. Moreover, it enables us to examine cases where reservoir permeability is heterogeneous. Under these circumstances, for heterogeneity indices beyond some critical value, fingering displacements are dominated by the permeability structure (the precise value of which should factor in inherent numerical dissipation present in the TOUGH2 simulator).

1. Introduction

Reinjection is a vital process in geothermal field management as it provides pressure support, reduces/mitigates subsidence, and at the same time, it is an environmentally friendly way for disposal of waste fluid (Kaya et al., 2011; Diaz et al., 2016). However, in situations where the reinjected fluid is hotter (and hence less viscous) than the fluid within the reservoir, this can lead to a viscous fingering instability. Here a spatially-uniform interface separating the two fluids can destabilise, leading to localised incursions (fingers) of the less viscous fluid into the more viscous one. This ultimately produces an interface with a highly-complex shape, and can be especially disruptive when it results in thermal breakthrough, affecting production and field operation (Kaya et al., 2011; Diaz et al., 2016).

The phenomenon of viscous fingering has been known for some time in the petroleum industry. The sharp interface here arises from difference in polarity between the injection fluid, water, and the oil. In oil reservoirs, it can be observed when the extraction rate is too high. Under these circumstances if there is a water layer lying beneath the oil, water can penetrate into the oil layer and be drawn into the well (Kidder, 1956). Fingering also occurs during the process of enhanced oil recovery (Sajjadi and Azaiez, 2012). By considering force balances across the oil-water interface Dietz (1953), also examined how gravity can lead to tonguing of the injected water underneath the oil within an inclined porous layer.

Saffman and Taylor (1958) were perhaps the first to apply a formal linear stability analysis to examine the general phenomenon of viscous fingering. Their analysis considered uniform unidirectional flow of two fluids of different viscosity, and they were able to determine criteria under which the planar interface between the fluids would destabilise and form fingers. In accompanying experiments they injected a less viscous fluid between two closely spaced glass sheets of a Hele-Shaw cell (which generates a regime that mimics porous flow) filled with a more viscous fluid, and confirmed the presence of the fingering instability.

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* Corresponding author.
E-mail addresses: s.zarrouk@auckland.ac.nz, sadiqzarrouk@gmail.com (S.J. Zarrouk).

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Nomenclature

A	Dynamic viscosity contrast
C_p	Specific heat capacity
c	Standard deviation
ϵ	Small parameter used in asymptotic expansions
γ	Square root of variance
H	Enthalpy
HI	Heterogeneity index
L	Length
l	Longitudinal wave number
λ	Thermal lag coefficient, correlation length scale
K	Thermal conductivity
k	Permeability
m	Wave number
M	Mobility ratio
n	Wave numbers of the periodicity in the x direction
μ	Dynamic viscosity
P	Reservoir pressure
p	Injected fluid pressure
q	Wave numbers of the periodicity in the y direction
Pe	Peclet number
ϕ	Porosity
Q	Volume flux rate
r	Radius measured away from well injection point
R	Log-mobility ratio
ρ	Density
σ	Standard deviation, growth rate
t	Time
T	Temperature
τ	Distribution mean
θ	Angle measured about central well injection point
U	Constant flow velocity
u	Fluid velocity
v	Variance
x, y	Spatial coordinates
ξ	Similarity coordinate
ζ	Perturbation amplitude

Subscripts

0	Initial, mean state
1	Injected fluid
∞	Native reservoir fluid
B	Basic state
c	Critical
D	Dimensional
r	Radial
θ	Azimuthal

Whilst the stability analysis of Saffman and Taylor (1958) deals with fluids separated by a planar interface, reinjection in reservoirs typically involves a point injection source, which results in a circular interface between the two fluids. As noted by Tan and Homsy (1987), this circular configuration leads to flows which change profile over time even before the onset of any instability. Through a linear stability analysis, again based on two different isothermal fluids with different viscosities, they showed that this has important implications for the growth rate of perturbations to the basic (i.e. non-perturbed) state. In the absence of any heat transfer to the solid phase of the porous medium Tan and Homsy (1987), also observed that there are two non-dimensional parameters that determine the stability of such a radial source flow. Firstly the Peclet number which measures the ratio of the rate of thermal advection to thermal diffusion, and depends only on reservoir properties and the mass injection rate. Secondly, the log-mobility ratio, which is the logarithm of the ratio of mobilities (or the difference between the logs of mobilities). It is worth noting that these types of linear stability analyses are only valid for the onset of instability, after which non-linear effects dominate. Predictions around the evolution of the instability at later times are therefore typically made using full numerical simulation.

Whilst viscous fingering is relatively well-studied in the context of petroleum engineering, fewer authors have addressed the subject of viscous fingering in relation to geothermal systems. In the geothermal reservoir context we are generally interested in the transport of two identical fluids, the viscosities of which are different due to their different temperatures. This is in contrast to the more commonly studied scenario where the difference in viscosities comes about because the two fluids being considered are different, though the approach is very similar.

Fitzgerald et al. (1994) used the geothermal simulator TETRAD to model injection of liquid into a porous layer containing superheated vapour. They were able to demonstrate the onset of fingering, however, the shape of the fingers predicted were shown to be sensitive to the finite difference scheme used (i.e. 5-point stencil compared with a 9-point stencil). This grid-orientation effect had previously been noted in similar numerical simulations. Peaceman and Rachford (1962) used a numerical scheme based on a finite difference formulation for modelling a petroleum reservoir. They found that grid-orientation error dampens fingering perturbations, due to the numerical dissipation introduced in approximating derivatives over the sharp fronts. The grid-orientation effect (GOE) is discussed more by Chen et al. (1993).

Tan and Homsy (1988) acknowledged the shortcomings of the finite difference method, and developed a numerical scheme using the Spectral Method. This approach allows for a high degree of accuracy in approximating spatial derivatives, and consequently reduced numerical dissipation. Sajjadi and Azaiez (2012) also developed a pseudo-spectral numerical method suitable for investigating a fingering stability in a simple model of heterogeneity.

Reservoir heterogeneity has a large influence on the unstable displacement patterns that occur during injection. Tan and Homsy (1992) modeled the heterogeneities as a stationary random distribution, and varied the correlation length scale and variance of the permeability perturbations. They show that there can be close coupling between fingering caused by the differences in viscosities (i.e. the non-zero log-mobility ratio), and that caused by channeling through the spatial structure of the heterogeneous permeability. The permeability structure determines the precise shape of the fingers, whilst the log-mobility sets their growth rate. Araktingi and Orr (1993) studied fingering in heterogeneous media by implementing a particle tracking algorithm to numerically model mass dissipation. A heterogeneity index clearly defines the conditions that give rise to each type of fingering, including a transition region. Riaz and Meiburg (2004) studied a periodic permeability field. A transition from flow dominated by viscous fingering to flow dominated by the spatial permeability distribution is identified.

We begin by investigating the onset criteria for this viscous instability. This is studied analytically with the theory of linear stability analysis. The resulting criteria were then applied to numerical models for verification using the TOUGH2 geothermal reservoir simulator.

2. Linear stability analysis

In this section we investigate the criteria for the onset of viscous instability using linear stability analysis. This analysis will later be supplemented with full numerical simulations using the TOUGH2 (Pruess et al., 1999) geothermal reservoir simulator (see Section 3).