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Technical Note

Analytical solution for a circular opening in a rock mass obeying a three-stage stress–strain curve

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1. Introduction

The displacements and stresses of the surrounding rock generally influence the stability of the underground engineering. A great number of research studies have attempted to predict the evolution law by examining different methods. Among these studies, an analytical method of elasto-plastic theory was the most widely used for the axisymmetric circular openings with the linear Mohr–Coulomb (M–C) criterion, as well as the nonlinear Hoek–Brown (H–B) criterion.[1]

Brown et al.[2] Wang,[3] and Carranza–Torres and Fairhurst[4] proposed analytical and numerical solutions based on elastic-perfectly-plastic models, respectively. At a later point in time, Sharan[5] pointed out the defects in the woprk of [2,3], and then derived the closed-form solutions of circular openings in H–B rock masses[5]. Park et al.[6] studied the displacements of the surrounding rock with four types of elastic strain distribution forms in the plastic region. However, only the solutions based on the generalized Hooke's law were found to be correct. Similarly, utilizing the assumption of the same elastic strain distribution in thick cylinders, Sharan derived analytical solutions for circular openings using a generalized H–B criterion.[7] Furthermore, Chen and Tonon[8] studied the analytical solutions of circular openings in generalized H–B rock masses. However, it was determined that the solution could be solved with a simple numerical procedure. Wang and Yin[9] derived the analytical solutions for spherical cavities excavated in elasto-brittle-plastic rock masses. Along with the analytical method, a numerical method was used for the more complicated engineering. Hajiabdolmajid et al.[10] proposed a CWFS model for the failure characteristics of brittle rock masses. This model was utilized to simulate the well-documented Mine-by Experiment, which demonstrated good accordance with the in situ conditions. Also, the strain-softening analysis of the circular openings was studied for the soft rock masses using both analytical and numerical methods.[11,12]

It is known that the above results are mainly dependent on the constitutive model. All of the former research studies regarding brittle rock masses were almost entirely based on elasto-plastic (EPM) and elasto-brittle-plastic (EBM) models. Furthermore, the majority of the laboratory tests determined that the brittle failure characteristics usually occur when the confining pressure is low. However, with the increase in confining pressure, brittle rock masses first of all usually experience a stable perfect-plastic stage before a brittle failure occurs. Moreover, the perfect plastic (even hardening) behavior would tend to appear once the confining pressure becomes very high. This is referred to as the brittle-ductile transformation process.[13] In regards to shallow underground engineering, an elasto–brittle–plastic model has the ability to effectively reflect the mechanical behavior of brittle rock masses. However, it cannot be overlooked that with increasing depth, the plastic bearing capacity is gradually enhanced, especially in

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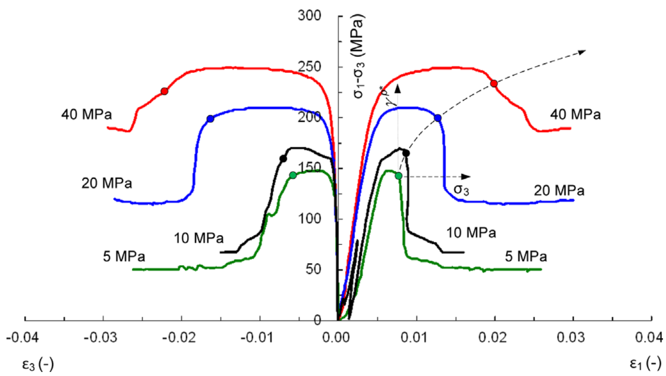

Fig. 1. Triaxial tests of brittle rock mass with various confining pressures.

deep underground engineering. Moreover, the deteriorated elastic parameters should also be considered in plastic rock masses.[14–17]

Therefore, this current study mainly deals with a constitutive model of brittle rock mass with consideration given to the plastic bearing capacity and deterioration of the elastic parameters. Moreover, the analytical solutions for circular openings, which were based on the EBPM proposed model, were also derived.

2. Proposed constitutive model

2.1. Triaxial tests

Fig. 1 illustrates the completed strain-stress curves of marble in Jinping, which had a depth of 2550 m, and was under different confining pressures.[18] It can be seen that the rock first experienced an elastic stage, and then the ideal plastic behavior occurred when it satisfied the yield function. In other words, the marble possessed a certain ideal peak plastic bearing capacity, and the plastic bearing capacity was enhanced with the increase of the confining pressure. As the deformation increased, the brittle failure occurred once the plastic strain reached a certain value. Then, the residual macro-plastic stage began.

Therefore, the mechanical behavior for brittle rock masses can be represented by three stages as follows: elastic stage, peak plastic stage, and residual plastic stage, as shown in Fig. 2. Additionally, it was determined that the elastic modulus and Poisson's ratio of the peak plastic stage may be different from those of the residual stage due to cracking. It was therefore obvious that the proposed EPBM combined the EPM and EBM with the brittle failure conditions.

2.2. Mechanical parameters

A great deal of previous research has determined that the Poisson's ratio has only a minimal influence on the deformation of rock masses. Therefore, it is assumed as a constant value in this current study. The calculated elastic moduli of the rock masses were 37.13 GPa, 46.33 GPa, 47.55 GPa, and 52.20 GPa for the confining pressures of 5 Mpa, 10 MPa, 20 MPa, and 40 MPa, respectively. Also, a mean value of 45.80 GPa was employed. Similarly, the Poisson's ratio was taken as 0.263. Meanwhile, the dilation angle could be easily calculated by using the maximum and minimum plastic strain, and the average values of 34.46° and 12.42° were taken for the peak plastic and residual plastic rock masses, respectively. It should be noted that the initial stage of the axial strain–stress curve with a confining pressure of 10 MPa showed a significantly nonlinear compression effect induced by the rock samples' unevenness. Therefore, an axial compression

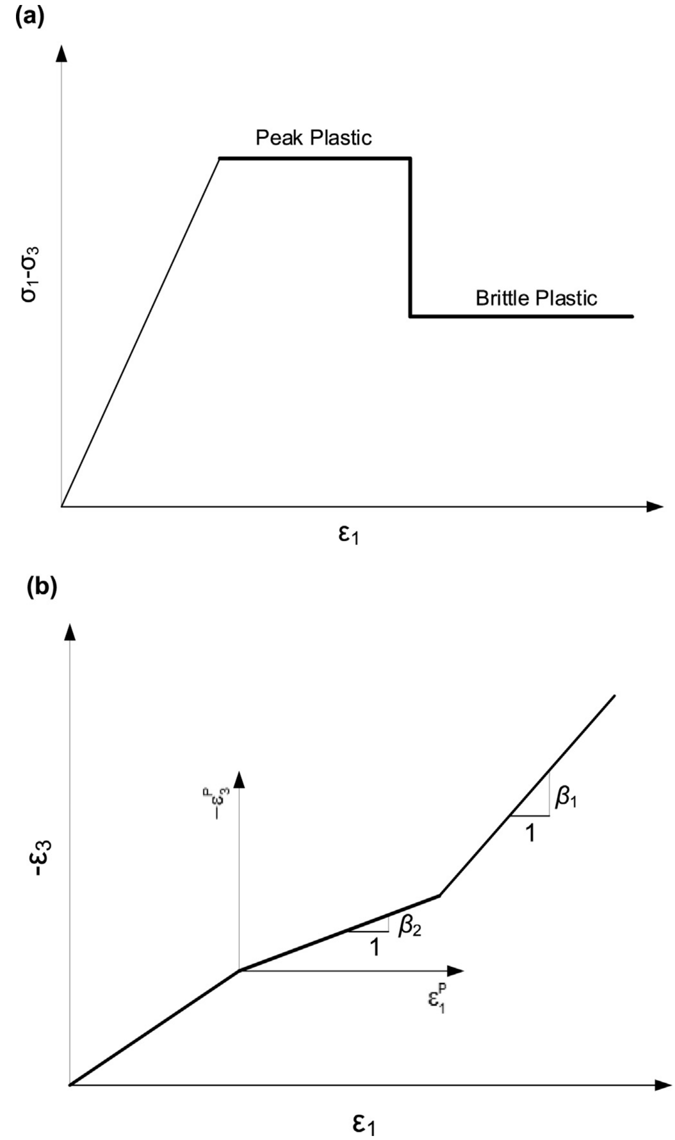

Fig. 2. Material behavior model for EPBM. (a) Stress–strain curve and (b) strain curve.

strain of -0.0015 was considered for this study.

The peak and residual strengths determine the state of the rock mass, and it is the foundation for calculating the plastic strain. The peak and residual strengths with linear Mohr–Coulomb criterion using the simplified triaxial tests curves is shown in Fig. 3. The peak cohesion c_p and friction angle φ_p were 35.53 MPa and 35.60°, respectively. Meanwhile, the residual cohesion c_r and friction angle φ_r were 7.38 MPa and 42.03°, respectively. The marble showed a brittle-ductile character with the increases in the confining pressure. The residual strength equaled the peak strength when the confining pressure σ_3=61.27 MPa. When $\sigma_3 > 61.27$ MPa, the proposed model showed a strain-hardening property. In order to ensure the correctness of the strain-hardening condition, high confining pressure tests with $\sigma_3 > 61.27$ MPa are recommended.

2.3. Critical failure condition

As stated previously, an irreversible shear plastic strain was employed to describe the critical brittle failure condition. When the brittle failure occurred, the shear plastic strain reached its ultimate value γ^{p^*}. Theoretically speaking, when the confining pressure was large enough, the mechanical behavior of the marble