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Technical note

Damage constitutive model based on energy dissipation for intact rock subjected to cyclic loading

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ABSTRACT

Rocks in underground engineering such as coal mining and tunnel excavating applications are usually loaded and unloaded repeatedly. The relationship between the strength and deformation of rocks under cyclic loading is the foundation for design and assessment of such scenarios. A new damage constitutive model based on energy dissipation was developed to describe the behaviour of rocks under cyclic loading in this article. First, the damage variable based on energy dissipation was introduced and the damage evolution equations of two typical rock types were calculated from the results of uniaxial cyclic loading tests. Second, the concept of compaction coefficient was proposed to describe the compaction degree and being used to amend the damage constitutive model obtained by Lemaitre strain equivalence hypothesis. Finally, the damage constitutive model under cyclic loading was established using a recursive method based on the amended damage constitutive model, and the fitted results of the models were compared to experimental data. Results showed that the values of damage variables increase exponentially with strains. The amended damage constitutive model can describe the degree of compactness of rocks accurately, and the damage constitutive model under cyclic loading has reasonable error in describing the behaviour of rocks under cyclic loading.

1. Introduction

A rock mass is a complex fractured geological medium containing numerous randomly distributed flaws, such as joints and cracks. Its constitutive model is the foundation for design and assessment in rock engineering and has been one of the key problems facing modern researchers.[1–5] Many researchers have made contributions to the understanding of the strength and deformability of rocks under uniaxial and conventional triaxial loading conditions. Constitutive models for rocks considering nonlinearity, anisotropy, rheology, and other properties have been established: these have laid solid theoretical foundations for engineering practice.[6–11] However, rocks are usually loaded and unloaded repeatedly during activities such as coal mining and tunnel excavation and support. Rock masses show hysteretic behavior under such conditions, and the strength and deformability are closely related to the stress state and loading history. Therefore, current constitutive models, under uniaxial and conventional triaxial loading conditions, cannot meet the requirements of engineering practice in more complex conditions.[2,12]

Researchers have investigated the strength and deformability of rocks under cyclic loading from the following two perspectives[3,13]: one involved building constitutive models for rock flaws such as cracks, joints, and interfaces under different loading conditions. For example, Fuenkajorn et al.[14] simulated the deformation of rock joints under cyclic loading using physical models. Yin et al.,[13] and Yu et al.[15] built constitutive models for cracks and joints in rocks under cyclic loading, respectively. David et al.[16] built a sliding crack model for nonlinearity and hysteresis in the uniaxial stress–strain curve of rock. The other approach has been phenomenological in which workers neglect the initiation, extension, and merging processes associated with microcracks or microvoids and focusing on describing the behavior of rocks as a whole. For example, Zhou et al.,[17] Song et al.[18] and Lee et al.[19] studied the failure properties and deformation mechanisms of rocks under cyclic triaxial loading. Liu et al.[20] and Liu et al.[21] studied the deformation properties of different rocks under cyclic loading.

The first approach can describe the strength and deformability of a known crack, joint, or other flaw in the rock. However, it still cannot be used to provide guidance for engineering practice where flaws are randomly distributed within the rock mass. Although the

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other approach cannot reveal the evolution mechanism of rock flaws, it can provide some guidance for engineering practice with regard to the deformability, strength, and strain energy evolution of rocks as a whole. Nonetheless, a constitutive model for rocks under cyclic loading, with convenient field application and high accuracy, has not yet been built.

Studies show that the deformation and failure of rocks are always accompanied by energy dissipation.[22,23] The energy dissipation is derived from damage within the rocks, so the damage and energy dissipation are correlated. This work first introduced a damage variable based on energy dissipation and calculated the damage evolution equations of two typical rocks based on cyclic loading tests. We then proposed a compaction coefficient to describe the compactness of a rock mass, and amended the constitutive model accordingly. Finally, the damage constitutive model for rocks under cyclic loading was established, and comparisons between fitted results from the models, and experimental results, were conducted.

2. Cyclic loading tests of rocks

2.1. Testing schemes and results

There were two kinds of rock samples (Groups 1 and 2): samples in Group 1 were taken from the roof of the No. 20307 coal face, Gaojialiang Mine, Inner Mongolia, China. Samples in Group 2 were taken from the roof of the No. 8704 coal face, Xinzhaoyao Mine, Shanxi Province, China. Rock samples in Group 1 were poorly cemented sandy mudstones with a pelitic texture, and a low strength. Rock samples in Group 2 were well cemented compact siltstones which were brittle and easily fractured. The rock samples were cut into standard cylindrical samples in accordance with the Chinese Standard "Standard Test Methods for Engineering Rock" (GB/T 50266-2013): the trimmed samples measured 50 mm in diameter and 100 mm in height.

Uniaxial loading tests were done first using an MTS815.03 servo-controlled testing machine to obtain the uniaxial compressive strength, σ_c, and the Young's modulus, E. Then, three samples from each group were selected and cyclic loading tests were conducted. The unloading stresses at the pre-peak stage were 40%, 60%, 80%, 90%, and 100% of the uniaxial compressive strength, and those at the post-peak stage were 90%, 70%, 50%, and 40% of the uniaxial compressive strength. Both the loading and unloading speeds were 5 mm/min. During the tests, the testing result that was greatly different from the others was removed. If the number of valid results of any test condition was less than three, we would test another sample until three similar results were obtained.

The mechanical properties of the rocks are shown in Table 1, and their stress–strain curves under cyclic loading are shown in Fig. 1. The average uniaxial compressive strength of siltstone (Group 2) was 72.75 MPa, about 8.6 times larger than that of the sandy mudstone (Group 1). The loading curve for the sandy mudstone decreases slowly throughout its post-peak stage, which meant that the rock still retained some carrying capacity. The areas of the hysteretic loops were large during the whole loading process, so there was a certain amount of energy dissipated, even in the pre-peak stage. The loading curve for the siltstone fell suddenly to zero after passing its peak, which meant that the rock experienced sudden failure. The areas of the hysteretic loops were small throughout the loading process, so there was little energy dissipated before failure. Moreover, the sandy mudstone broke into many small pieces with much dust produced, while the siltstone broke into several large blocks with fewer smaller fragments present.

Table 1
The mechanical properties of the rocks.

Rocks	Density (g cm^{-3})	Young's Modulus (GPa)	Poisson's ratio	Uniaxial compressive strength (MPa)
Sandy mudstone	2.32	0.84	0.22	8.47
Siltstone	2.66	8.52	0.205	72.75

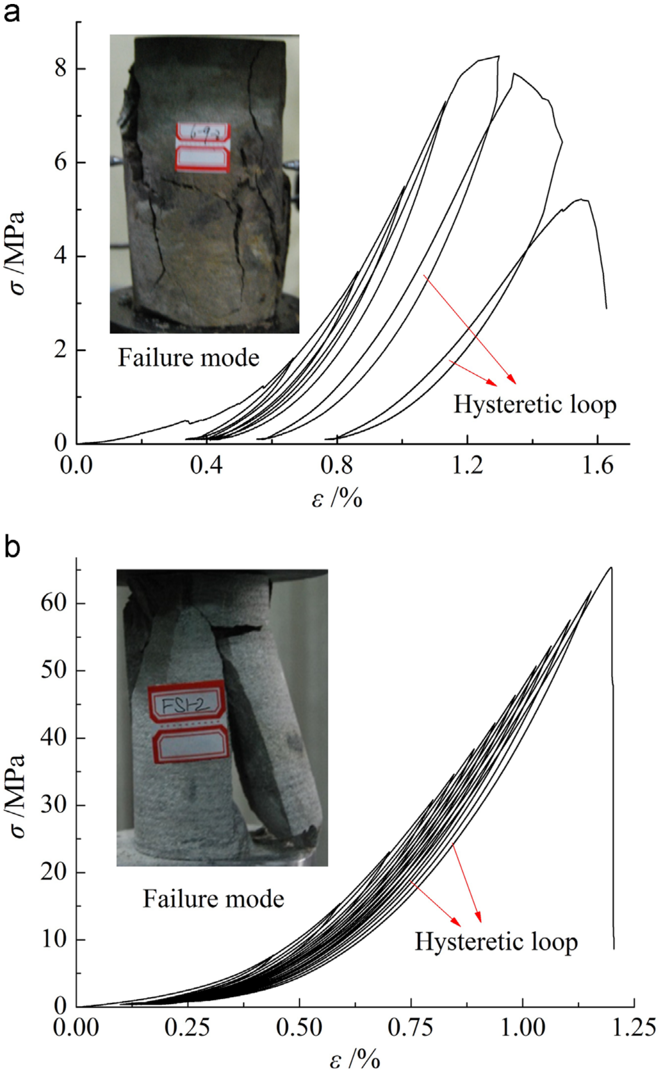

Fig. 1. Stress–stain curves under cyclic loading and the failure modes of rocks: (a) Sandy mudstone and (b) siltstone.

2.2. Damage variable and evolution equations

2.2.1. Damage variable

The concept of a damage variable was proposed in continuum damage mechanics to describe the extent of the development of flaws such as joints and cracks in a continuum. The damage variable for rock has been defined using many parameters, such as joint spacing, Young's modulus, yield stress, wave velocity, and acoustic emission event count. Studies show that the energy dissipation is derived from damage in rocks and that the damage and energy dissipation are correlated. Therefore, some researchers have suggested defining the damage variable of rocks on the basis of energy dissipation and hold the view that it can accurately reflect the changes in the mechanical properties of rocks.[23,24] Jin et