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Technical Note

Estimation of mean trace length by setting scanlines in rectangular sampling window

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1. Introduction

The behavior of a rock mass including mechanical properties and hydraulic characteristics is strongly influenced by the size of discontinuities. In practical rock engineering, the size of three-dimensional (3-D) discontinuities cannot be measured directly. Therefore it is necessary to calculate the trace length that indicates the condition of 3-D rock discontinuities on sampling plane. In a finite window, the trace lengths of traces whose both ends are observable can be measured, but the trace lengths of others having one or both ends censored cannot be measured. Hence the accurate value of mean trace length cannot be determined easily; however, the likely value can be estimated based on probability statistics.[1]

At present, there are many different mean trace length estimation methods. The three widely used methods can be summarized as follows: (a) sampling traces that intersect a line drawn on the exposure, referred to as scanline survey,[2,3] (b) sampling traces within a finite area (usually rectangular or circular in shape) on the exposure, referred to as area (or window) sampling[4–7] and (c) sampling traces that intersect a circle drawn on the exposure, referred to as circle sampling.[8] In the window sampling method, the intersection may occur in three ways depending on the position of the discontinuity: (1) both ends censored, (2) one end censored and (3) both ends observable in the window. However, the calculated result depends on the quantity of traces in the three ways. Further, the limitation of rectangular window sampling method is finite spatial coverage. Therefore, the traditional window sampling method is not very reliable, and the calculated result is not a very stable one.

Mauldon[9] proposed a method of estimation of mean trace length in arbitrary convex windows. In their study, Yang et al.[10] demonstrated that the tangent circle method is stable and reasonable. Chen et al.[11] improved rectangular window sampling method for a rectangular window and Wu et al.[12] showed how to perform integrations given in the equations of rectangular window sampling method using discretized values. The present study synthesizes the advantages of both the scanline survey and window sampling methods and proposes a new method to estimate the mean trace length by arranging scanlines in a rectangular sampling window. Further, rectangular window sampling method is shown to be a special case of the proposed method and the reasons for the disadvantages of rectangular window sampling method are identified. The proposed method can produce a more accurate mean trace length. The proposed method is shown to be satisfactory by five simulated cases and a practical engineering situation.

2. Mathematical model

The proposed method of estimation of mean trace length by setting scanlines in a rectangular sampling window is derived

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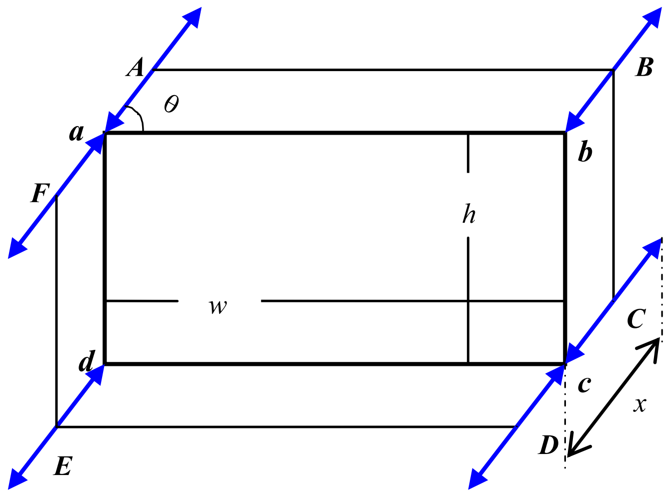

Fig. 1. Intersection of discontinuities with a rectangular window. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

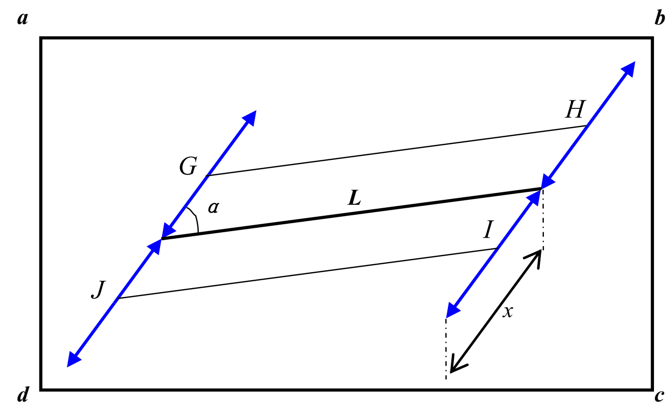

Fig. 2. Sketch of the new method. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

from the sampling window method proposed by Kulatilake. Consider traces in a rectangular sampling window *abcd* of width w and height h (Fig. 1). Assume that the midpoints of the traces are uniformly distributed in two-dimensional (2-D) space. Let λ be the mean density of the trace midpoints per unit area. The blue double-headed arrows indicate the traces having length x and apparent dip θ. Assume that the trace length x and the apparent dip θ are statistically independent of each other. In addition, each of the traces with midpoints A, B, C, D, E and F has one end just touching the edge of the window. For a trace to intersect the window *abcd*, the midpoint of the trace must lie in the zone of polygon *ABCDEF*. Then, the area of the polygon *ABCDEF* (S) is given as below:

$$\begin{aligned} S &= S_{ABCDEF} = S_{abcd} + S_{ABba} + S_{dcDE} + S_{FadE} + S_{bBCc} \\ &= wh + \frac{1}{2}wx\sin\theta + \frac{1}{2}wx\sin\theta + \frac{1}{2}hx\cos\theta + \frac{1}{2}hx\cos\theta \\ &= wh + wx\sin\theta + hx\cos\theta \end{aligned} \tag{1a}$$

Let us assume that the trace length x ($x \in [0, \infty)$) is described by a probability density function $f(x)$, and the probability density function of the apparent dip θ ($\theta \in [0, 90]$) is $g(\theta)$. Let n be the expected number of discontinuities with $x\epsilon$ $(x,x+dx)$ and $\theta\epsilon$ $(\theta, \theta+d\theta)$ intersecting the window. Then,

$$n = \lambda S f(x) g(\theta) dx d\theta = \lambda(wh + wx\sin\theta + hx\cos\theta) f(x) g(\theta) dx d\theta \tag{1b}$$

The total number of traces (N) in the window can be obtained by integrating n over x and θ as follows:

$$\begin{aligned} N &= \lambda \int_0^\infty \int_0^{90} S f(x) g(\theta) dx d\theta \\ &= \lambda \int_0^\infty \int_0^{90} (wh + wx\sin\theta + hx\cos\theta) f(x) g(\theta) dx d\theta \end{aligned} \tag{1c}$$

Because $\int_0^\infty f(x)dx = 1$, $\mu = \int_0^\infty x f(x) dx$, $\int_0^{90} g(\theta) d\theta = 1$, $\int_0^{90} \sin\theta g(\theta) d\theta = E(\sin\theta)$ and $\int_0^{90} \cos\theta g(\theta) d\theta = E(\cos\theta)$, the above Eq. (1c) can be reduced to

$$N = \lambda[wh + \mu w E(\sin\theta) + \mu h E(\cos\theta)] \tag{1d}$$

where, μ is the mean trace length that we want.

The scanline L is an arbitrary scanline in the finite rectangular window *abcd*. The length of the scanline L is l (Fig. 2). The blue double-headed arrows indicate traces with length x and relative apparent dip α with the scanline L. Let us assume that the trace length x and relative apparent dip α are statistically independent of each other. Further, each of the traces with midpoints G, H, I and J has one end nearly touching the endpoint of scanline L. For a trace to intersect the scanline L, the midpoint of the trace must lie in the zone of the parallelogram *GHIJ*. Let S_L be the area of parallelogram *GHIJ* given by

$$S_L = xl\sin\alpha \tag{2a}$$

Assume that the trace length x ($x \in [0, \infty)$) is described by a probability density function $f(x)$, and the probability density function of the relative apparent dip α ($\alpha \in [0, 90]$) is $g(\alpha)$. The expected number of discontinuities (n_L) with $x\epsilon$ $(x,x+dx)$ and $\alpha\epsilon$ $(\alpha,\alpha+d\alpha)$ intersecting the scanline L can be given by

$$n_L = \lambda S_L f(x) g(\alpha) dx d\alpha = \lambda xl\sin\alpha f(x) g(\alpha) dx d\alpha \tag{2b}$$

The total number of traces intersecting the scanline L (N_L) in the window can be obtained by integrating n_L over x and α as given below:

$$\begin{aligned} N_L &= \lambda \int_0^\infty \int_0^{90} S_L f(x) g(\alpha) dx d\alpha \\ &= \lambda \int_0^\infty \int_0^{90} (xl\sin\alpha) f(x) g(\alpha) dx d\alpha \end{aligned} \tag{2c}$$

Because $\int_0^\infty f(x)dx = 1$, $\mu = \int_0^\infty x f(x) dx$, $\int_0^{90} g(\alpha) d\alpha = 1$ and $\int_0^{90} \sin\alpha g(\alpha) d\alpha = E(\sin\alpha)$, the above Eq. (2c) can be reduced to

$$N_L = \lambda\mu l E(\sin\alpha) \tag{2d}$$

where, μ is the mean trace length that we want.

From Eqs. (1d) and (2d), we get

$$R_L = \frac{N_L}{N} = \frac{\mu l E(\sin\alpha)}{\mu[wE(\sin\theta) + hE(\cos\theta)] + wh} \tag{3}$$

where $R_L = N_L/N$ is the ratio of the expected number of traces intersecting the scanline L.

Further, the mean trace length (μ) can be derived from Eq. (3) as

$$\mu = \frac{whR_L}{lE(\sin\alpha) - R_L[wE(\sin\theta) + hE(\cos\theta)]} \tag{4}$$

Assume that the number of scanlines is m, and scanline L_i ($i\epsilon$ $[1, m]$) is the scanline L with length l_i. Further, the total number of traces intersecting the scanline L_i is N_i, and R_{Li} and α_i denote the ratio of the expected number of traces intersecting the scanline L_i and the relative apparent dip with scanline L_i, respectively. Then,