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A unified micromechanics-based damage model for instantaneous and time-dependent behaviors of brittle rocks

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ABSTRACT

This paper presents a unified micromechanics-based damage model for instantaneous and time-dependent inelastic behaviors of brittle rocks subjected to compressive stresses. The constitutive model is formulated in a combined homogenization/thermodynamics framework. The inelastic deformation is induced by frictional sliding along closed cracks, and strongly coupled with damage evolution by crack growth. Material degradation is described by a scalar-valued internal damage variable that is decomposed into two parts: an instantaneous part induced by applied stresses and a time-dependent part by subcritical cracking due to stress corrosion. Based on the system free energy determined with the Mori–Tanaka homogenization scheme, we propose a Coulomb-type friction criterion, which serves simultaneously as the yielding function and plastic potential, implying the use of an associated flow rule. An instantaneous damage criterion based on the conjugated force associated with the damage variable and a time-dependent damage criterion in terms of progressive evolution of microstructure are introduced. For the latter, an efficient computational algorithm is explored to solve numerically the strain history-dependent integration. As the first phase of validation, the proposed model is finally applied to simulate two typical brittle rocks, Dagangshan diabase and Xiangjiaba sandstone.

1. Introduction

Constitutive modelling of instantaneous and time-dependent nonlinear behaviors of brittle rocks is one of the main research issues in rock mechanics and rock engineering community. Concerning instantaneous behaviors of brittle rocks, the nucleation, growth and coalescence of microcracks is largely viewed as the main source of material deterioration and final failure [2,3,1]. In order to describe this deterioration process, the concept of continuum damage was introduced [6,4,5]. Within the framework of irreversible thermodynamics, a considerable body of phenomenological damage models have been proposed (an exhaustive list of which is beyond the scope of this introduction). The main advantage of these phenomenological models consists that they can be easily implemented into computer codes and applied to engineering analyses. However, the assumptions made and fitting parameters involved therein are not clearly based on physical mechanisms related to dissipative microcracking. To amend this shortcoming, some micromechanics-based constitutive models have been developed and applied successfully to model induced damage in brittle media [7–15]. On the other hand, theoretical analyses such as [11,14,15] have shown that damage-friction coupling analyses can interpret and simulate quite satisfactorily a large spectrum of instantaneous mechanical phenomena of brittle rocks, for example, non-linearity of stress–strain relations, deterioration of elastic properties, significant volume dilatation, damage-induced material softening, *etc*.

Time-dependent inelastic deformation and damage by crack growth may produce simultaneously in geomaterials like rocks and concrete. Creep deformation in brittle rocks is generally less pronounced than that in soft ones like sedimentary rocks and salt rocks [16–19]. For brittle rocks, such time-dependent behaviors are commonly interpreted as the consequence of the effect of stress corrosion. In that context, the subcritical cracking theory postulates that crack growth may occur even for damage driving force lower than its critical limit [20–22]. In practice, the subcritical cracking mechanism has been successfully applied to explain and simulate time-dependent properties of rock-like brittle materials [23,24].

In the last few decades, constitutive models for creep behaviors have been widely investigated within the theoretical framework of

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either viscoelasticity or viscoplasticity [25–31]. These two kinds of phenomenological models show obvious disadvantages for further coupling extensions. On the other hand, Shao et al. [32] and Pietruszczak et al. [33] proposed a particular framework to describe creep deformation in geomaterials, in which time-dependent plastic deformations are postulated as the consequence of progressive microstructure evolution of the material. However, mainly focusing on time-dependent damage, the model ignored instantaneous damage caused by increasing applied stresses. It is also noticed that various experiments confirmed the existence of a good correlation between time-dependent inelastic deformation and crack growth [21,22,26]. In this sense, the multi-scale damage-friction coupling models have the potential of being extended to take into account time-dependent deformation and damage by subcritical cracking.

The objective here is to develop a unified damage model to describe simultaneously the instantaneous and time-dependent nonlinear behaviors of brittle rocks. Based on the Mori–Tanaka homogenization scheme [34], a micromechanical damage-friction coupling model is first formulated within the framework of irreversible thermodynamic for stresses-induced creep deformation and damage evolution. To this end, a Coulomb-type friction criterion is formulated in terms of the local stress tensor that contains a back-stress term, and a strain energy release rate-based damage criterion is used. An appropriate damage evolution law in terms of microstructure change due to subcritical cracking is proposed and framed by the method in Shao et al. [32] and Pietruszczak et al. [33]. Moreover, full damage-friction coupling analyses are performed in order to determine damage and friction multipliers involved in the consistency conditions. Moreover, a fast explicit algorithm for strain history-dependent integration is explored to improve computational efficiency [32]. Finally, the proposed model is applied to typical brittle rocks, Dagangshan diabase and Xiangjiaba sandstone. Comparisons between numerical predictions and experimental data are presented.

Throughout the paper, the following notion on tensorial product of any second order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ will be used: $(\boldsymbol{A}\otimes\boldsymbol{B})_{ijkl}=A_{ij}B_{kl}$. With the second order identity tensor $\boldsymbol{\delta}$, the usually used fourth order identity tensor $\mathbb{I}$ and the fourth order hydrostatic projector $\mathbb{J}$ are expressed in the components' form as $I_{ijkl}=\frac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$ and $J_{ijkl}=\frac{1}{3}\delta_{ij}\delta_{kl}$, respectively. The fourth order deviatoric projector $\mathbb{K}=\mathbb{I}-\mathbb{J}$ is then obtained. Moreover, the fourth-order tensors $\mathbb{J}$ and $\mathbb{K}$ have the properties: $\mathbb{J}:\mathbb{J}=\mathbb{J}$, $\mathbb{K}:\mathbb{K}=\mathbb{K}$, $\mathbb{J}:\mathbb{K}=\mathbb{K}:\mathbb{J}=0$.

2. Constitutive formulations over the REV

For upscaling analyses, a representative elementary volume (REV) is generally required to represent the idealized microstructure of the material. For microcracked brittle rocks, the relevant REV is taken to be a matrix-inclusions system composed of an isotropic matrix (elasticity tensor $\mathbb{C}^s$) and a large number of randomly distributed and oriented microcracks. The goal is to construct macroscopic constitutive equations based on microstructure information and local material laws.

2.1. Strain decomposition

All microcracks in the REV are in fact discontinuities embedded in the solid matrix. Therefore, the total strain over the REV can be decomposed into two parts

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^s+\boldsymbol{\varepsilon}^c, \tag{1}$$

where the first part on the right hand side denotes the elastic strain related to the deformation of the matrix and the second part $\boldsymbol{\varepsilon}^c$ is the inelastic strain due to displacement discontinuities between crack surfaces. According to strain decomposition (1), the macroscopic stress $\boldsymbol{\sigma}$ is given as follows:

$$\boldsymbol{\sigma}=\mathbb{C}^s:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^c). \tag{2}$$

When the solid matrix is linearly isotropic, the elasticity tensor $\mathbb{C}^s$ takes the classic form $\mathbb{C}^s=3k^s\mathbb{J}+2\mu^s\mathbb{K}$ with k^s and μ^s being the bulk and shear moduli of the rock matrix, respectively.

2.2. Damage decomposition

In order to make direct use of the basic Eshelby's solution, all microcracks are assumed to penny-shaped in geometry and characterized by its unit normal vector $\boldsymbol{n}$ and aspect ratio $\epsilon=c/a\ll 1$ with a and c denoting the average radius and the half opening of cracks, respectively. It is then possible to express mathematically the volume fraction φ of a family of microcracks as $\varphi=\frac{4}{3}\pi a^2 c\mathcal{N}=\frac{4}{3}\pi\epsilon d$ where $\mathcal{N}$ is the number of cracks per unit volume of all microcracks in the considered family, and $d=\mathcal{N}a^3$ is the crack density parameter.

In brittle materials, two kinds of cracks-related damage, *i.e.*, stress-induced instantaneous damage and time-dependent damage by subcritical cracking, should be taken into account. In other words, damage due to nucleation and growth of microcracks can be decomposed into two parts:

$$d=\omega+\varsigma, \tag{3}$$

where ω and ς denote instantaneous damage and time-dependent damage, respectively. For later use, we cast Eq. (3) into its incremental form

$$\mathrm{d}d=\mathrm{d}\omega+\mathrm{d}\varsigma. \tag{4}$$

For simplicity, assume that $\mathrm{d}\varsigma=0$ during short-term mechanical loading.

2.3. Free energy and state laws

For open microcracks, damage by cracking is viewed as the unique energy dissipative source. According to the linear homogenization method [13,14,35,36], the inelastic strain $\boldsymbol{\varepsilon}^c$ can be related linearly to the macroscopic strain $\boldsymbol{\varepsilon}$ that are applied onto the boundary of the REV

$$\boldsymbol{\varepsilon}^c=\varphi\mathbb{A}^c:\boldsymbol{\varepsilon}, \tag{5}$$

where $\mathbb{A}^c$ denotes the global strain concentration tensor of the crack phase.

Under isotropic assumption and by applying the Mori–Tanaka (MT) scheme [34,37], Eq. (5) takes the following explicit expression [15]:

$$\boldsymbol{\varepsilon}^c=\left(\frac{\eta_1 d}{1+\eta_1 d}\mathbb{J}+\frac{\eta_2 d}{1+\eta_2 d}\mathbb{K}\right):\boldsymbol{\varepsilon}, \tag{6}$$

where η_1 and η_2 are two constants only function of Poisson's ratio ν^s of the matrix phase. For penny-shaped microcracks, one has $\eta_1=\frac{16}{9}\frac{1-(\nu^s)^2}{1-2\nu^s}$ and $\eta_2=\frac{32}{45}\frac{(1-\nu^s)(5-\nu^s)}{2-\nu^s}$. Insertion of Eq. (6) into Eq. (2) leads to the homogenized elasticity tensor $\mathbb{C}^{\text{hom}}$

$$\mathbb{C}^{\text{hom}}=\mathbb{C}^s:(\mathbb{I}-\varphi\mathbb{A}^c)=\frac{1}{1+\eta_1 d}3k^s\mathbb{J}+\frac{1}{1+\eta_2 d}2\mu^s\mathbb{K}. \tag{7}$$

Under small strain assumption and in isothermal process, the free energy, noted W, can be expressed as