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Numerical simulations of propagation, bifurcation and coalescence of cracks in rocks

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ABSTRACT

Propagation process of pre-existing flaws in brittle rock materials are simulated using peridynamic theory, in which it is assumed that particles in a continuum interact with each other across a finite distance. Since damage is incorporated at the level of these two particle interactions in peridynamic theory, localization and fracture of materials occur as a natural outgrowth of equations of motion and constitutive models. Numerical simulations of notched semi-circular bend (NSCB) tests and mix mode fracture in a tension–shear rock sample are performed. The effects of array of multiple pre-existing flaws on the propagation process in rock-like materials subjected to uniaxial tensile loads are investigated. The propagation process of macroflaws and microflaws in rock-like materials subjected to uniaxial tensile loads are simulated.The failure modes of rock sample containing the different array of pre-existing flaws are studied. It is found that the present numerical results obtained from peridynamic theory are in good agreement with the previous experimental and numerical results.

1. Introduction

Rock masses contain various types of pre-existing flaws. The propagation process of these pre-existing flaws under compressive and tensile loads are significant in the study of rock engineering. Extensive study has been done on crack propagation in the different rock-like materials under uniaxial compression in 2D experimental studies[1–10] and numerical studies[11–16]. However, the previous experimental and numerical studies mainly focus on the propagation process of the pre-existing flaws in rock-like materials under uniaxial compressive loads, while the experimental and numerical studies on the propagation process of the pre-existing flaws in rock-like materials under tensile loads is very limited. The main reason is that it is very difficult to investigate crack propagation process in rock-like materials under tensile loads in experimental studies. In fact, the failure of rock masses is not only due to compressive loads, but also due to tensile loads. For example, the excavation of tunnels may lead to the occurrence of tensile stress in the surrounding rock masses. The failure of the surrounding rock masses around tunnels occurs when the value of tensile stress is more than that of the uniaxial tensile strength of rock masses. Moreover, since the uniaxial tensile strength of rock masses is smaller than the uniaxial compressive strength of rock masses, the failure of rock masses subjected to tensile loads is easier than that of rock masses subjected to compressive loads. Therefore, it is important to numerically investigate the propagation process of the pre-existing flaws in rock-like materials under tensile loads.

Many numerical methods are put forward to model the propagation process of flaws by researchers. The finite element method (FEM) has been applied to investigate crack growth and coalescence. In the finite element-based method, singular crack-tip elements are frequently encountered.[17] Because of the crack tip stress singularity, external fracture criterion must be introduced to determine the crack coalescence and bifurcation, and the problem of the crack nucleation is still not solved.[18] In order to overcome the above difficulties, the extended finite element theory[19] is proposed to simulate the propagation of cracks. Although many problems of cracks are solved by virtue of the extended finite element method, bifurcation criterion must still be introduced when displacement is discontinuous, and when multiple crack interaction and bifurcation are involved. Besides, a series of difficulties are encountered for the problem of the three-dimensional cracks by the XFEM. In order to solve the problems of interactions among cracks and bifurcation phenomenon of multiple cracks, meshless methods are developed.[20] For example, the propagation of cracks can be simulated by Smooth Particle Hydrodynamics (SPH). However, tensile instability problems are encountered in SPH.[21] The molecular dynamics can be applied to model the propagation of

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cracks. However, the molecular dynamics has some shortcomings, such as the longer computational time and the lower computational efficiency.[22] The phase-field theory can also be applied to model the fracture of the brittle material, in which any external criteria does not need to be adopted.[23]

In order to overcome the aforementioned shortcomings, peridynamic theory (PD), which is a meshless numerical method based on the nonlocal concept, is introduced to model bifurcation and coalescence of cracks. It is assumed that particles in a continuum interact with each other across a finite distance, its formulation is by integral equations rather than partial differential equations.[24] Therefore, the peridynamic method (PD) can be applied to model the problems of continuous or discontinuous displacements.[25–27] The peridynamic method (PD) needs not to make use of any external criteria. Moreover, it breaks through the singularity problem of crack tips, it has the virtue of meshless methods and molecular dynamics method, and it avoids the limitations of calculation dimension of molecular dynamics method. Based on the peridynamic method, the bifurcation and coalescence phenomenon of cracks can be taken place spontaneously. Growth velocity and branching angle of the cracks can be modeled correctly, especially for the brittle material.[28]

Both the peridynamic theory and the phase-field theory can be applied to model the fracture of the brittle material, and they do not need any exterior criterion. However, there are some differences between the peridynamic theory and the phase-field theory. The differences are summarized as follows: (1) The peridynamic method is a meshless particle method, which is based on a nonlocal theory. In peridynamic method, materials are discretized into some particles with mass. While the phase-field theory is a mesh element method, in which materials are discretized into some meshes. (2) For the peridynamic theory, the governing equations are in integral form, whereas for the phase-field theory, the governing equations are of partial differential form. (3) The physical meaning of s is different. In peridynamic method, the parameter s is the bond stretch, whereas in the phase-field theory, the parameter s is an additional continuous field.

2. Basic theory

The peridynamic theory (PD) was proposed by Silling.[29] The peridynamic equation of motion is given by[29]:

$$\rho(x)\ddot{u}(x,t) = \int_H \left(t_{(k)(j)}\left(u_{(j)}-u_{(k)},x_{(j)}-x_{(k)},t\right) - t_{(j)(k)}\left(u_{(k)}-u_{(j)},x_{(k)}-x_{(j)},t\right)\right)dH + b(x,t) \tag{1}$$

where $\rho(x)$ is the mass density, H is a neighborhood of the material point $\delta/\delta_s = 1.0$, $b(x,t)$ is a prescribed body force density field of the material point $x_{(k)}$, which represents the external force per unit reference volume square, $t_{(k)(j)}(u_{(j)}-u_{(k)},x_{(j)}-x_{(k)},t)$ and $t_{(j)(k)}(u_{(k)}-u_{(j)},x_{(k)}-x_{(j)},t)$ are respectively the force density vector of the material point $x_{(k)}$ and $x_{(j)}$.

For an isotropic and elastic material, the following formulation can be expressed as[29]:

$$t_{(k)(j)}\left(u_{(j)}-u_{(k)},x_{(j)}-x_{(k)},t\right) = \frac{1}{2}f\left(u_{(j)}-u_{(k)},x_{(j)}-x_{(k)},t\right) = \frac{1}{2}\mu\left(x_{(j)}-x_{(k)},t\right)c_{(k)}\left(s_{(k)(j)}-\alpha\Delta T\right) \tag{2}$$

$$t_{(j)(k)}\left(u_{(k)}-u_{(j)},x_{(k)}-x_{(j)},t\right) = \frac{1}{2}f\left(u_{(k)}-u_{(j)},x_{(k)}-x_{(j)},t\right) = -\frac{1}{2}\mu\left(x_{(k)}-x_{(j)},t\right)c_{(j)}\left(s_{(j)(k)}-\alpha\Delta T\right) \tag{3}$$

where $f(u_{(j)}-u_{(k)},x_{(j)}-x_{(k)},t)$ and $f(u_{(k)}-u_{(j)},x_{(k)}-x_{(j)},t)$ are respectively the pairwise force function between the material point $x_{(k)}$ and $x_{(j)}$, α is the coefficient of thermal expansion of the material, ΔT is uniform temperature change; $s_{(k)(j)}$ and $s_{(j)(k)}$ are respectively the bond stretch between material point $x_{(k)}$ and $x_{(j)}$, it is defined by:

$$s_{(k)(j)} = s_{(j)(k)} = \frac{\left|x_{(j)}+u_{(j)}-x_{(k)}-u_{(k)}\right| - \left|x_{(j)}-x_{(k)}\right|}{\left|x_{(j)}-x_{(k)}\right|} \tag{4}$$

where $\mu(x_{(j)}-x_{(k)},t)$ and $\mu(x_{(k)}-x_{(j)},t)$ are respectively a history-dependent scalar-valued function between the material point $x_{(k)}$ and $x_{(j)}$. The function μ it is given by

$$\mu\left(x_{(j)}-x_{(k)},t\right) = \begin{cases} 1 & s\left(t',x_{(j)}-x_{(k)}\right) < s_0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where s_0 is the critical stretch for bond failure; for plane stress problem, when critical stretch value is obtained, it is defined as

$$s_0 = \sqrt{\frac{4\pi G_0}{9E\delta}} \tag{6}$$

where E is the elastic modulus;δ is a positive number, which is called the horizontal radius; the work G_0 required to break all bonds per unit fracture area is then found from

$$G_0 2\int_0^{\delta}\int_x^{\delta}\int_0^{\cos^{-1}\frac{z}{\xi}}\left(\frac{1}{2}c\xi^2 s_0^2\right)d\phi d\xi dz = \frac{1}{4}cs_0^2\delta^4 \tag{7}$$

where $c_{(k)}$ and $c_{(j)}$ are respectively a "spring constant" of material point $x_{(k)}$ and $x_{(j)}$, which are described respectively as follows:

$$c_{(k)} = \frac{9E_{(k)}}{\pi\delta^4} \tag{8}$$

$$c_{(j)} = \frac{9E_{(j)}}{\pi\delta^4} \tag{9}$$

where $E_{(k)}$ and $E_{(j)}$ are the elastic moduli of the material points $x_{(k)}$ and $x_{(j)}$.

In order to model the fracture, a notion of local damage[30–32] at a point is introduced, which is defined as

$$\varphi\left(x_{(k)},t\right) = 1 - \frac{\int_H \mu\left(x_{(k)},t,\epsilon\right)dV_\epsilon}{\int_H dV_\epsilon} \tag{10}$$

where $\varphi(x_{(k)},t)$ is local damage value of any point $x_{(k)}$, its ranges are $0 \le \varphi(x_{(k)},t) \le 1$,with 0 representing virgin material, 1 representing the complete disconnection of a point from all of the points with which it initially interacted.

Substituting Eqs. (8) into (2), the following expression can be