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An implicitly coupled hydro-geomechanical model for hydraulic fracture simulation with the discontinuous deformation analysis

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A B S T R A C T

In this article we present and verify an algorithm for modeling hydraulic fracturing in complex fracture geometries. The method is built upon an existing model which uses the discontinuous deformation analysis for simulation of rock mechanics and upon a finite volume fracture network model for simulation of compressible fluid flow in fractures. Improvements are made to the fluid, contact and coupling components of the existing algorithm to increase its accuracy and stability. The model is successfully benchmarked against the analytical solution for the opening profile of an internally pressurized Griffith fracture, and against the semi-analytical solution for the pressure and opening profile of a bi-wing hydraulic fracture propagating under conditions of no leakoff and no toughness. Additionally, the model is verified through comparison with the results of a hydraulic fracturing experiment that examined propagation of a high-viscosity fracturing fluid in an impermeable medium.

1. Introduction

Hydraulic fracturing is a technique widely used in the energy industry for increasing the permeability of natural gas and geothermal systems. In a typical hydraulic fracturing operation, highly pressurized fluid is injected into low-permeability rock formations to create fractures in the rock which act as channels for fluid flow. As hydraulic fracturing has become more widespread, a variety of computational tools have been developed to aid in the hydraulic fracturing design process. Among these are a suite of analytical and semi-analytical solutions [1,2] based around the classic Perkins-Kern-Nordgren (PKN) [3,4] and Khristianovich-Geertsma-DeKlerk (KGD) models [5,6], which can predict the properties of a bi-wing fracture with fixed geometry. Other tools include the pseudo-3D (P3D) and planar-3D (PL3D) models [7], which allow for bi-wing fracture modeling in layered rock media. Although these tools are widely used, a common limitation of all of these methods is that they can only model single propagating fractures confined within a pre-defined vertical plane.

As an alternative, a few methods are currently under development which model hydraulic fracturing within complex fracture geometries [8–11]. Underlying each of these models are individual procedures for simulating rock mechanics, fluid mechanics and fracture mechanics. For rock mechanics, the majority of these methods use either the explicit finite element method (FEM) or the discrete element method (DEM), both of which are based on explicit methods for time integration. Similarly, in these applications the fluid mechanics procedure is also solved using explicit techniques. Although these methods can successfully model fracturing in complex geometries, the use of explicit methods for time integration requires the use of small time steps and mesh grids for computational stability, greatly increasing the computational cost of the overall simulation. As an alternative, this work develops a fully implicit algorithm for hydraulic fracture modeling based on the discontinuous deformation analysis (DDA) for the rock mechanics, which is hereafter referred to as the HFDDA. Hydraulic fracture modeling based on the DDA was first proposed in [12–14], which applied the coupling scheme developed in [15] to examine bi-wing fracture propagation and two-dimensional fracture propagation in the presence of background fractures. In this article, the hydraulic fracturing algorithm proposed in [12–14] is enhanced and verified against two analytical solutions for hydraulic fracture growth in an infinite medium and against an experimental analysis of hydraulic fracture growth in an impermeable material.

2. Governing equations

2.1. Hydraulic fracture modeling

Hydraulic fracturing of a reservoir begins after a well has been drilled, and steel pipe and well casing have been inserted to protect the overlying formations. The well casing is perforated in the targeted reservoir, allowing the fracturing fluid to be injected and

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to contact the gas bearing rock. Eventually, the rate of injection will exceed the rate of fluid absorbed by the rock formation, causing the fluid pressure to build. Once the fluid pressure has reached the breakdown pressure of the formation, the highly pressurized fluid will create new fissures and open pre-existing ones in the impermeable rock. Modeling of this process requires consideration of a few key mechanisms. First, fluid mechanics must be considered, either within the fractures alone, or as a combination of flow within the fractures and flow within the rock matrix. Second, a method needs to be selected for modeling the deformation of fracture walls as a response to fluid pressure. Generally, it is assumed that the deformation of the rock matrix is linear elastic, such that linear elastic fracture mechanics (LEFM) can be used to describe the stress and displacement distributions around the propagating fracture. Finally, a mechanism needs to be included to account for the formation of new fractures within the system. In typical hydraulic fracturing models, fracturing of new rock is considered to occur in Mode I only, although in hot dry rock geothermal modeling, Mode II fracturing is considered the dominant mechanism [16]. The complexity of the model and its solution will be directly dependent on the dimensionality of the original problem, the number of processes modeled, and the level of detail being considered.

2.2. Fluid flow model

The fluid-flow module in the HFDDA is derived using a finite volume fracture network form of the conservation equations for fluid mass and momentum and follows the method applied in [12,15]. The network comprises fluid nodes connected by fractures (Fig. 1), which form the spaces between rock blocks. For a fluid node i, conservation of mass is given by the equation

$$\frac{\partial}{\partial t}(\rho_i V_i) = \sum_{j=1}^{k} Q_{ij} + Q_{L,i} + C_i \tag{1}$$

where k is the number of nodes j connected to node i, ρ_i is the fluid density, V_i is the volume of node i, C_i is the mass injection rate, Q_L is the leakoff into the fracture formation, and Q_{ij} is the flow rate along fracture ij. For one-dimensional (1D) flow, the flow rate along the fracture may be expressed by Poiseuille's Law using the well-known cubic law

$$Q_{ij} = \frac{-\rho_{ij} w_{ij}^3 h_{ij}}{12\mu_{ij} L_{ij}}\left(p_i - p_j\right) = -\alpha_{ij}\left(p_i - p_j\right) \tag{2}$$

where w_{ij} is the fracture width, L_{ij} is the length, h_{ij} is the depth (equal to unity for plane-strain conditions), μ_{ij} is the fluid viscosity (assumed constant), p_i is the pressure in node i, ρ_{ij} is the density in fracture ij, and α_{ij} represents the transmissivity of the fracture. The fracture density and fracture widths may be approximated as the average of the densities and widths at nodes i and j, respectively.

Leakoff of the fluid may be expressed using Carter's leakoff coefficient (C_L) and the area opened along the walls of each fracture using the expression

$$Q_{L,i} = -\rho_i \sum_{j=1}^{k} \frac{C_L}{\sqrt{t - t_{0,ij}}} \frac{h_{ij} L_{ij}}{2} = \rho_i Q_{LV,i} \tag{3}$$

where $Q_{LV,i}$ is the volumetric rate of leakoff from node i, t is the simulation time and t_0 is the time at which fluid first arrives at the node from fracture ij. For compressible flow, fluid density is a function of pressure by the definition

$$\rho_i = \rho_0\left(1 + c_f(p_i - p_0)\right) \tag{4}$$

where c_f is the fluid compressibility, and ρ_0 and p_0 are a reference density and pressure, respectively. Substitution of Eqs. (2)–(4) into Eq. (1) and application of a forward discretization in time yields the final form of the fluid flow equation for each node, given as

$$\left(c_f\rho_0\left(V_i^{t+\Delta t} - \Delta t Q_{LV,i}{}^{t+\Delta t}\right) + \Delta t \sum_{j=1}^{k} \alpha_{ij}^{t+\Delta t}\right) p_i^{t+\Delta t} - \left(\Delta t \sum_{j=1}^{k} \alpha_{ij}^{t+\Delta t} p_j^{t+\Delta t}\right)$$
$$= \left(\rho_0\left(1 - c_f p_0\right)\left(\left(V_i^t\right) - \left(V_i^{t+\Delta t} - \Delta t Q_{LV,i}{}^{t+\Delta t}\right)\right)\right) + \left(c_f\rho_0 V_i^t\right)p_i^t + \Delta t C_i^{t+\Delta t} \tag{5}$$

Note that this equation differs from the similar equations in [15] and [13,14], as it accounts for leakoff and uses a fully implicit method to describe the change in pressure resulting from the change in volume of the node itself. For all of the nodes in the fracture system, Eq. (5) can be rewritten as the matrix system

$$\mathbf{A}\mathbf{p} = \mathbf{B} \tag{6}$$

where $\mathbf{p}$ is the vector of unknown pressures at time $t+\Delta t$, $\mathbf{A}$ is the matrix of coefficients for the unknown pressure vector, and $\mathbf{B}$ includes the terms on the right hand side of Eq. (5). In Eq. (6), for a given rock geometry, all of the terms in $\mathbf{A}$ and $\mathbf{B}$ are known except for the average fracture density ρ_{ij}, which is part of α_{ij} in the matrix $\mathbf{A}$, and is dependent on the fluid pressures by the relationship in Eq. (4). Thus for a given fracture geometry determined by the DDA, Eq. (6) must be solved iteratively to reconcile the average densities ρ_{ij} in $\mathbf{A}$ at time $t+\Delta t$ with the pressures at time $t+\Delta t$.

One difficulty that arises when solving Eq. (6) is that there is no constraint on the pressures in the system to be positive. If the void volume at a fluid node increases beyond the volume of fluid contained within it, the pressure at that node will become negative, a situation which physically would not happen. To remedy this problem, if a node has a negative pressure in the HFDDA after the solution of Eq. (6), a zero pressure boundary condition is assigned to that node and Eq. (6) is resolved. However, to ensure conservation of mass, the amount of fluid that flows into these boundary nodes must also be calculated (Fig. 2). Returning to Eq. (5), when a zero pressure condition is assigned to a node, the volume of fluid in the node at the new time ($V_i^{t+\Delta t}$) becomes

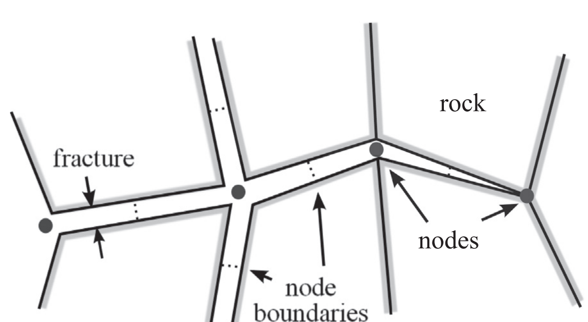

Fig. 1. Example of fluid and rock system showing various components within the HFDDA.

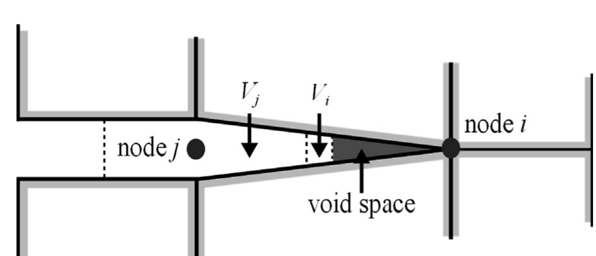

Fig. 2. Geometry demonstrating the void space and fluid volume at fluid nodes where zero pressure bounds are assigned.