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Extension of discontinuous deformation analysis and application in cohesive-frictional slope analysis

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ABSTRACT

This paper extends the discontinuous deformation analysis (DDA) by using an additional evaluation of edge-to-edge contact, with the aim that it can be used to accurately model the failure behaviour of joints dominated by both cohesion and interface friction angle. The original DDA can deal well with the effects of interface friction angle. However, when cohesion exists, DDA results often show an inscrutable behaviour, i.e. a slope may be unstable even if the cohesion is much greater than the theoretical value required for its critical stability. After many detailed investigations and validations, joint contact treatment was found to be the key reason why the original DDA cannot simulate the cohesive material accurately, in which every edge-to-edge contact is treated as two vertex-to-edge contacts that may have different contact states associated with different cohesion treatments. In order to solve this problem, an additional contact type determination process for an edge-to-edge contact was added into the original computer code to avoid the unreasonable situation when two contact states exist in one joint. Several examples were performed to illustrate the accuracy of the modified code and a real landslide case was analysed by using the improved DDA to estimate the shear strength on the interface. Our results show that the improved DDA can simulate the failure of cohesive-frictional material accurately.

1. Introduction

Discontinuous deformation analysis (DDA)[1,2] is a discrete numerical method that was developed for computing large deformation and large displacement in a discontinuous blocky system. DDA introduces a unified format accounting for not only the translation, rotation and deformation of an individual block but also movement forms such as sliding and opening along block boundaries, having the advantages of both the distinct element method (DEM) and the finite element method (FEM) [3]. Since the novel formulation and the numerical code of DDA were presented, DDA draws more and more attention. Many modifications and improvements to the original formulas have been proposed to overcome some of its limitations [4–11] and make it more efficient, suitable and practical to engineering computations, for rockfalls [12–15], landslides [16–24], tunnel [25–27], blast [28–30], dynamic block [31–35] and others [36–40]. In addition, a number of high-profile projects were studied by the DDA, e.g. the Three Gorges Dam project in China [41,42], Pueblo Dam in Colorado [43], the Yerba Buena tunnel portal, San Francisco [44], Norway's Gjovik Olympic Cavern [45], Israel's Masada National Monument [46,47] and so on.

The behavior of a blocky system depends on the strength of block boundaries, including cohesion c, interface friction angle φ (or friction coefficient $\tan\varphi$) and tension strength σ_t. In the original DDA code, the Mohr–Coulomb failure criterion is applied in block interface and a penalty technique is used to prevent interpenetration between blocks. In order to validate the accuracy of DDA, many researchers compared the results between DDA and other methods, including (i) analytical methods, (ii) other numerical methods and (iii) laboratory and field investigations. In these validation cases, a single block on an incline is the simplest and the most efficient one. If the strength of interface is only from friction, the errors are generally lower than 1% [31–33,39]. However, once the strength of joint contains cohesion, the accuracy will reduce sharply, i.e. the original DDA cannot simulate the failure behavior of cohesive material accurately. That is why even

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one validation example for cohesive material does not exist in more than 100 validation cases referenced by a review paper [48].

Literature review shows the original DDA often provides an underestimated factor of safety when applied to cohesive materials. In order to improve the accuracy of DDA, there are several studies related to cohesive failure behaviour, in which, strength degradation method is often used. This method is on the basis of that the strength of interfaces can depend on displacement [11,17] or velocity [34,35]. Therefore, using the displacement- or velocity-dependent shear strength can improve the accuracy. However, the functions c(*displacement* or *velocity*) and φ(*displacement* or *velocity*) requires specific data from experiment or user defined which are usually difficult to obtain or decide. So, the failure behavior simulation of cohesive material in DDA is still an unsolved problem when applying the Mohr–Coulomb joint failure criterion to common cohesive materials.

The purpose of this paper is to present an improved DDA that can accurately simulate the failure behavior of cohesive-frictional material. Accurate calculation of failure depends on the calculation of contact force in interface. In the original DDA, one edge-to-edge contact is treated as two vertex-to-edge contacts that may have different contact states, and once one of the vertex-to-edge contacts fails, half of the cohesion force between the joint is improperly removed. This unreasonable situation, when one joint has two contact states along with two treatments of cohesion, is the reason why the original DDA cannot simulate the cohesive material accurately. In this study, an additional evaluation of edge-to-edge contact state is added into the original DDA code to avoid the situation when two contact states coexist in one edge-to-edge contact. A series of analytical solutions and DDA results are presented to validate the accuracy of the improved DDA.

2. Theory of DDA

2.1. Basic theory and time discretization

DDA is based on the minimization of total potential energy of a block assembly. It is formulated based on block theory where each block can move and deform independently, and the interaction between blocks is idealized by contact springs. Zhang et al. [23,24] described the general theory of DDA in detail as follows:

For a two-dimensional problem, each block of arbitrary geometry has six degrees of freedom, among which three components are rigid body motion terms and the other three are constant strain terms. So the deformation variable of block i can be written as

$$D_i = (u_0 \;\; v_0 \;\; r_0 \;\; \varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy})^T \tag{1}$$

where u_0, v_0 are the translations of the block centroid (x_0, y_0) along the x and y axes, r_0 is the rigid rotation around (x_0, y_0), and $(\varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy})$ are the normal and shear strains of a block at (x_0, y_0). The displacement $U=(u, v)$ at any point (x, y) of a block can be represented by:

$$U = TD_i \tag{2}$$

where T is the displacement transformation matrix, defined by

$$T = \begin{bmatrix} 1 & 0 & -(y-y_0) & x-x_0 & 0 & \frac{(y-y_0)}{2} \\ 0 & 1 & x-x_0 & 0 & y-y_0 & \frac{(x-x_0)}{2} \end{bmatrix} \tag{3}$$

DDA computations take the first order approximation of the displacement function, which represents the constant stress and strain at any arbitrary point within the block. The block is assumed to be elastic, and the shear resistance at the boundary is assumed to follow the Mohr–Coulomb yield criterion when each pair of blocks make contact with each other.

A system of blocks is formed by many blocks through contacts among block edges and displacement constraints on each individual block. For the block system, the simultaneous equilibrium equations are similar to the system equations in the finite element method (FEM), i.e. Hamilton's principle and minimized potential energy. The following equation can be established:

$$M\ddot{D} + C\dot{D} + KD = F \tag{4}$$

where D, $\dot{D}$, $\ddot{D}$ are the matrices of displacement, velocity and acceleration, respectively; M is the mass matrix, C is the damping matrix, K is the stiffness matrix. F is the external force matrix. The damping matrix C in Eq. (4) can be rewritten as follows in terms of viscosity η and mass matrix M:

$$C = \eta M \tag{5}$$

The physical meaning of viscosity η is the damping of the rock itself, the viscosity of air around the rock surfaces and the vegetation on the surface of a rock slope. In this study, no viscous damping is introduced, i.e. $\eta=0$.

The kinematic Eq. (4) is solved by Newmark's β and γ method by using parameters $\beta=0.5$ and $\gamma=1.0$ as shown by the following equations,

$$D_{n+1} = D_n + \Delta t_n \dot{D}_n + \frac{\Delta t_n{}^2[(1-2\beta)\ddot{D}_n + 2\beta\ddot{D}_{n+1}]}{2} \tag{6}$$

$$\dot{D}_{n+1} = \dot{D}_n + \Delta t_n[(1-\gamma)\ddot{D}_n + \gamma\ddot{D}_{n+1}] \tag{7}$$

where subscript n denotes the nth calculation step. From Eqs. (6) and (7), the initial displacement D_n at calculation step $n=0$ is assumed to be 0 because the updating Lagrange descriptions are used in the analysis. Then

$$\ddot{D}_{n+1} = \frac{D_{n+1} - D_n - \Delta t_n \dot{D}_n - ((\Delta t_n{}^2(1-2\beta)\ddot{D}_n)/2)}{\beta \Delta t_n{}^2} \tag{8}$$

Substitution of Eq. (8) into Eq. (4) yields the global form.

The algebraic equation for the increase in displacement is solved at each time step by the following equation:

$$\tilde{K} \cdot D_{n+1} = \tilde{F}_{n+1} \tag{9}$$

where $\tilde{K}$ is the effective stiffness matrix and $\tilde{F}$ is the effective force matrix. Assuming that a block system consists of n blocks, we have

$$\tilde{K} = \begin{bmatrix} K_{11} & K_{12} & \ldots & K_{1n} \\ K_{21} & K_{22} & \ldots & K_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ K_{n1} & K_{n2} & \ldots & K_{nn} \end{bmatrix}, D = \begin{bmatrix} D_1 \\ D_2 \\ \ldots \\ D_n \end{bmatrix} \text{ and } \tilde{F} = \begin{bmatrix} F_1 \\ F_2 \\ \ldots \\ F_n \end{bmatrix}$$

where D_i, $F_i (i=1, 2,\ldots,n)$ are 6×1 submatrices, D_i is the deformation variable of block i, F_i is the load distributed to the six degrees of freedom of block i, K_{ij} $(i, j=1, 2,\ldots,n)$ is a 6×6 submatrices, K_{ii} is relevant to the material properties of block i and K_{ij} is defined by the contact between blocks i and j.

2.2. Contact mechanism

Blocks in a blocky system can contact with each other at their boundaries. For a two-dimensional problem, there are three possible contact types, vertex-to-vertex, vertex-to-edge, and edge-to-edge. In the original DDA method, an edge-to-edge contact is treated as two vertex-to-edge contacts. Hence, only two contact types, vertex-to-edge and vertex-to-vertex, are calculated in the original DDA code. This process reduces computational cost and simplifies DDA implementation.

Penalty technique is used in the original DDA to prevent interpenetration between blocks. In the original DDA code, when