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An improved method for numerically modeling the minimum horizontal stress magnitude in extensional stress regimes

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ABSTRACT

The minimum horizontal stress magnitude, S_{hmin}, is a crucial input parameter for a variety of subsurface engineering applications. Several methods, such as the general poro-elastic model, the uni-axial strain model and the concept of frictional equilibrium can be used to simulate S_{hmin}, whereby the general poro-elastic model is most commonly used due to its capability to account for tectonic strain in order to match existing stress measurements. If stress measurement data is unavailable this paper introduces a pre-stressing procedure for 3D numerical Mechanical Earth Models that combines the poro-elastic model with the frictional equilibrium model to provide lower bounds for S_{hmin} in extensional stress regimes. Assuming common friction coefficients of μ in the range of 0.57 to 1, the necessary horizontal strain can be calculated to limit horizontal stress magnitudes of the whole model domain or of only certain calibration layers by frictional failure. For layers with Poisson's ratios smaller or larger than 0.25, S_{hmin} magnitudes being too low or too high (as predicted by the uni-axial strain model) can thus be prevented. The presented concept is tested for two case studies and the modeling results show that the combination of the poro-elastic model with the frictional equilibrium model can provide a good match to the measured data, even if it is assumed that the calibration data is not available. It is concluded that the combination of the two deformation mechanisms can produce a more physically appealing stress profile and hence may more accurately simulate the sense and relative magnitude of layer-to-layer stress contrasts. In addition, the numerical modeling approach presented can match observations on the near surface variation of the ratio $k=S_{hmean}/S_v$.

1. Introduction

In recent years Mechanical Earth Models (MEMs), which in general represent a compilation of rock strength and stress data, have become a standard practice for many sub-surface engineering applications [1,2]. Especially numerical MEMs utilizing the finite element method have proven to be valuable for pre-drilling stress mapping applications [2], geomechanical risk assessment for CO_2 sequestration [3–5] and reservoir depletion analyses [6,7]. Other applications include mud weight calculations to ensure stable drilling conditions, fault stability analysis, hydraulic fracture operations and reservoir site characterization. A crucial input and calibration parameter for these numerical MEMs is the minimum horizontal stress, S_{hmin}, especially in extensional stress regimes (i.e. $S_{hmin} < S_{hmax} < S_v$) such as the East Texas Basin, the Gulf of Mexico, and the Central North Sea Graben where S_{hmin} is the least principal stress. Since direct stress measurements of S_{hmin} are not always available, a variety of different techniques based on the linear poro-elastic model (including the uni-axial strain model) [*e.g.* [8–11]] and the frictional equilibrium model [12] have been proposed to estimate the magnitude of S_{hmin}. However, these techniques are based on a variety of assumptions and simplifications [13,14], and thus have limited applicability to calculate the horizontal stress in a general fashion. The 3D *in-situ* state of stress is the result of a combination of various time dependent physical processes including rock diagenesis, compaction, heating and cooling, poro-elastic deformation and stress relaxation [14]. Rock heterogeneity, anisotropic material properties and complex three-dimensional geological structures add to the level of complexity. Furthermore, analytical models [15,16] show that the spherical geometry of the Earth explains the relative increase of the mean horizontal stress vs. the vertical stress at shallow depths.

The challenge for many scientists and engineers is how to realistically simulate *in-situ* stress magnitudes in a numerical model that are representative of the long-term genesis of a geological volume. In order to simulate spatially continuous *in-situ* stress magnitudes including non-linear material behavior and heterogeneous structures with complex geometries (faults, stratigraphic layers) 3D numerical MEMs most commonly utilize finite

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element analysis (FEA) based on the equations of linear porous-elasticity. The initial setup of the numerical model including gravitational loading requires a stress initialization procedure (also termed pre-stressing) wherein the modeled stresses as a result of gravitational compaction reach a state of equilibrium. A common procedure to establish stress equilibrium due to gravitational loading utilizes a boundary condition setting where only gravity is acting and the model boundaries are constrained such that only in-plane displacements (*i.e.* roller boundary conditions) are allowed (Fig. 1a; it should be noted that the geometry in Fig. 1 is based on a Cartesian coordinate system where the model is represented by a rectangular cuboid volume and it is assumed that for small model dimensions the curvature of the Earth is not affecting model results) [*e.g.* [4,17–20]]. This set of boundary conditions represents the uni-axial strain model and the resulting state of stress is termed "initial state of stress" or reference state of stress [19,21] and is free of any tectonic contribution. This initial state of stress is then used for subsequent modeling steps, which may include lateral kinematic boundary conditions to simulate the tectonic contribution to the state of stress [*e.g.* [19]] (Fig. 1b).

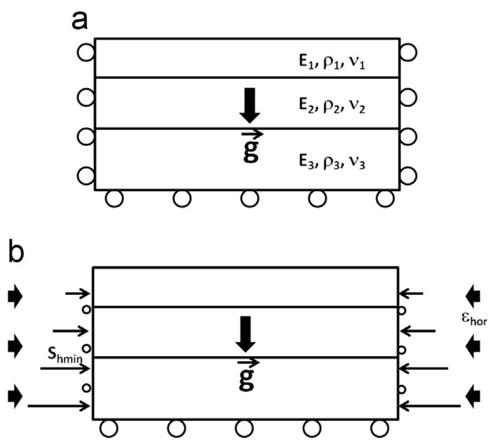

Fig. 1. Boundary conditions for 2D numerical MEMs to simulate realistic stress magnitudes. (a) The first step termed 'gravitational pre-stressing' utilizes a boundary condition setting where only gravity is acting and the model sides are constrained such that only in-plane displacements are allowed (rollers). (b) After reaching gravitational equilibrium a constant tectonic strain ε_{hor} can be added to the model, which results in changing horizontal stresses with depth.

A common practice in a numerical MEM application is to assume steady-state thermal conditions and to calibrate the resulting stress prediction against existing stress measurements by an addition of lateral strain (termed poro-elastic model; Fig. 1b) [*e.g.* [10]]. Strain resulting from thermal loading, such as during the burial and uplift history of the geologic structure, can be also added by including the reservoir temperature history. For most studies this information and the associated creep parameters are difficult to obtain and thus steady-state thermal conditions are often assumed. While this represents a significant, but often necessary simplification, steady-state thermal conditions still permit the analysis of subsequent thermal loading induced by processes such as steam injection.

If ε_{hor} is small enough such that $S_{hmin} < S_{hmax} < S_v$ an extensional stress regime is resembled. If no horizontal strain is applied in the second step the uni-axial strain model is reproduced. The necessity of adding a lateral strain component poses challenges when stress calibration measurements are not available. If the strain history is unknown, the question of how much strain is to be added to a MEM becomes important. In this context, the observation that significant parts of the Earth's crust are in frictional equilibrium [12,14], and hence the application of an appropriate

Table 1
Different methods to predict S_{hmin} magnitudes and their respective strengths and limitations.

Ability to	Method		
	Uni-axial strain model	**Poro-elastic model**	**Frictional equilibrium model**
Follow trends in pore pressure	As each method follows the same general equation (equations 1-3) in which the pore pressure is a constant input parameter, each method has the same sensitivity to trends in pore pressure.		
Predict layer to layer variations of S_{hmin} due to variations in			
(a) Coefficient of friction (μ)	Insensitive to μ.	Insensitive to μ.	Only parameter[a] responsible for different S_{hmin} magnitudes across layer boundaries.
(b) Poisson's ratio (ν)	Only parameter[a] responsible for different S_{hmin} magnitudes across layer boundaries.	One of two key input parameters[a] (ν, E) responsible for different S_{hmin} magnitudes across layer boundaries.	Insensitive to ν.
(c) Young's modulus (E)	Insensitive to E.	One of two key input parameters[a] (ν, E) responsible for different S_{hmin} magnitudes across layer boundaries.	Insensitive to E.
Predict unequal horizontal stress	Not possible	Possible due to 3D nature of equations.	Not possible.
Predict S_{hmin} for different stress regimes	Only applicable for extensional stress regimes.	Applicable for all stress regimes.	Applicable for extensional stress regimes. Limited applicability for strike-slip stress regimes: S_{hmax} needs to be known.
Predict lower bounds on S_{hmin}	Is able to provide lower bounds for all settings except where extensional strains are present.	Difficult, since estimate of lateral tectonic strain is dependent on many variables/data, especially for case studies where no stress calibration data exists.	If μ is known: able to provide lower bounds for majority of rock formations according to Byerlee's law [26].
Obtain model parameters from geophysical logs	All model parameters can be obtained from geophysical logs: S_v from integrated density log, ν from sonic log.	Model parameters S_v, ν, E can be obtained from geophysical logs: S_v from integrated density log, ν, E from sonic log; tectonic strain has to be estimated.	μ cannot be obtained directly from logs. It can be derived by empirical relations based on log measurements [13,27].
Obtain model parameters from laboratory measurements on cores	ν can be obtained from uni-axial strength tests.	ν, E can be obtained from uni-axial strength tests; tectonic strain has to be estimated.	μ can be obtained from triaxial strength tests.
To be calibrated to actual stress measurements	Rarely matches actual field data [10,11,22,23] and empirical correction factors are often used.	Can provide good match [10,11].	Can provide good match [14].

[a] Assuming constant pore pressure and that lateral density variations are not present.