International Journal of Rock Mechanics & Mining Sciences 64 (2013) 132–138

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Fracturing of an Euler–Bernoulli beam in coal mine pillar extraction

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ARTICLE INFO

Article history:
Received 27 February 2012
Received in revised form
14 June 2013
Accepted 2 August 2013
Available online 20 September 2013

Keywords:
Coal mine
Pillar extraction
Snook
Bending moment
Euler–Bernoulli beam
Euler strut

ABSTRACT

The paper models the first stage of the process of pillar extraction in a coal mine. The problem of understanding how a coal mine roof collapses after secondary cutting of the supporting pillars to create small supporting snooks is considered. The fracture of the roof is considered when a set of snooks have failed and the roof must support itself between two pillars. Models that account for the relative importance of the overburden weight on the roof and the compressive stresses in the roof are examined using a simple strut and beam theory.

1. Introduction

The need to return to old mine workings and extract a significant fraction of the remaining coal is becoming of increasingly economic importance and viability. In the current methods of secondary mining, the existing large pillars of coal that have been left to support the roof are cut away and, as the work proceeds back up the mine, the roof is left to collapse. This collapse needs to occur in a safe and controlled manner and this is done by carefully cutting the pillars into smaller structures, called snooks, so that these fail in a manner that lets the roof fall slowly and the working area remains free from falling material. Understanding how the snooks fail and how the roof fractures is central to creating a safe working environment. Determining how much of the pillars can be cut and hence how small the snooks can be made determines the fraction of coal that can be taken by this secondary mining method.

There are two stages in pillar extraction [1]. In the first stage several pillars are extracted and replaced by snooks. The load on each snook is increased as more and more pillars are extracted. When a snook fails, it fails violently. After about seven or eight pillars have been extracted the snooks fail. The roof layer cracks, primarily by bending induced tension. Eventually the roof and overburden collapse. In this first stage the roof layer can be modelled as a beam which is clamped at each end at a pillar. After the first collapse of the roof, the failure will occur at more frequent intervals. In this second stage of pillar extraction, one end of the roof layer will be supported by a snook and the other end by a pillar which is the working face. The end at the snook will no longer be clamped because of the failure of the surrounding rock and may be modelled as simply supported or hinged.

In this paper we consider the first stage of pillar extraction. Each end of the roof layer is clamped at the pillar and a number of snooks fail. The roof remains in place and supports the local weight of the rock and the large compressive stresses due to the overburden. We shall investigate the fracture of the roof and then summarise our insight into the behaviour.

2. Mathematical model of roof fracture

The problem that we consider is how the roof fractures when a number of snooks fail. There is considerable previous work on analyzing such situations which is well reviewed in [2]. For an elastic beam with no joints and no axial compressive stress and clamped at pillars at each end, the solutions for the maximum stress and maximum beam deflection have been obtained using the beam equation [3,4]. Vertical tensile fractures form at the pillars and the beam becomes simply supported

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1365-1609/$ - see front matter
http://dx.doi.org/10.1016/j.ijrmms.2013.08.001

at the pillars. The maximum stress is now at the midspan which is found to be greater than the previous stress at the pillars. This leads to fracturing at the midspan and other points of the beam [5,6].

To analyse the problem of fracturing of the roof we consider the case where the snooks fail completely and the roof has to be supported by the strength of the rock. The rock can be taken to have various constitutive relations but here we take the response to be elastic and we consider the roof to be made of horizontal layers of rock each acting as a beam. An elastic beam with no joints or fractures after an excavation is extremely rare [4]. Our model will describe the first stage in the fracture of the beam and the transition from a continuous elastic beam to a voussoir beam model cut with joints and fractures [2,4,7–9]. We assume that the roof extends for a long way along the roadway and hence that the behaviour only varies with distance across the roadway from one remaining pillar to another. We also assume that the only displacement is vertical. In practice this beam will be loaded from three main sources. Firstly the compressive stresses due to the overburden will create compressive stresses horizontally along the beam, secondly the beam will have to support its own weight and thirdly the beam will have stresses transferred to it from adjoining layers of rock. As an initial investigation we ignore the transfer of stresses from adjoining layers. Underground layers of rock tend to separate on deflection [10,4] and such transfer of stress may be small in cases where the beam may have displaced sufficiently to detach from layers above it. Hence our idealised problem is to investigate buckling of a beam that has a horizontal stress acting along its length due the overburden stress and that has to support its own weight.

This problem is therefore a combination of the conventional Euler–Bernoulli beam theory and the theory of the Euler strut. The beam is taken to be made of homogeneous material of thickness h. We take the mine to be at a depth H. We can expect the horizontal stress in the rock to be $k\rho gH$ where k is the lateral stress coefficient, ρ is the average density of the rocks above the depth H and g is the acceleration due to gravity [1,11,12]. The derivation of this result is briefly outlined in Section 5. We assume that this stress creates a horizontal force exerted at both ends of the beam which we denote by $P = hb(k\rho gH)$, where b is the breath of the beam. Moreover, the roof is subjected to its weight per unit length $q = \rho ghb$ that we assume equally distributed along the beam on all the surface. In practice we might wish to make a simple extension to partly allow for other layers above the roof on this beam by adding s to q and still denoting it by q where s is the applied surface traction per unit length vertically downwards on the top surface of the beam. Both s and q have the same effect on the displacement and enter the Euler–Bernoulli beam equation in the same way [13].

We will use the notation and conventions of Segel and Handelman [13]. The coordinate axes are defined in terms of the undeformed beam. The x_1- and x_2-axes are along the axes of principal moment of inertia of the cross-section of the beam with the x_1-axis vertically downwards. The x_3-axis is horizontal and passes through the centroid of each cross-section. The origin of the coordinate system is at the centroid of the cross-section of the left end of the beam. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are directed along coordinate axis. The problem is illustrated in Fig. 1.

We use analysis relevant to both an Euler–Bernoulli beam and an Euler strut. The displacement of the beam from the horizontal position, x_3, vertically downwards in the positive x_1-direction, is denoted by $w(x_3)$. An outline of the derivation of the differential equation for $w(x_3)$, has been given by Segel and Handelman [13] for a beam which is simply supported or hinged at each end. For a simply supported beam the displacement and its second derivative vanish at each end. We will consider a beam which is clamped at each end so that the displacement and its first derivative vanish at each end. The derivation of Segel and Handelman can be adapted to a beam for which each end is clamped instead of simply supported and it can be verified that the same differential equation for the displacement is obtained. Here we will give a plausible justification for this differential equation.

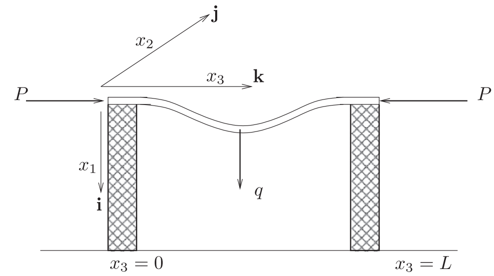

Fig. 1. A combined beam and strut.

Consider first the problem in which the horizontal axial force P does not act on the beam. Then the bending moment about the origin, $\mathbf{M}$, which is in the $\mathbf{j}$-direction, has magnitude

$$M = EI\frac{d^2w}{dx_3^2}, \tag{1}$$

where E is the Young's modulus of the roof rock and I is the second moment of area about the x_2-axis given by

$$I = \int_{-\frac{b}{2}}^{\frac{b}{2}} dx_2 \int_{-\frac{h}{2}}^{\frac{h}{2}} x_1^2\, dx_1 = \frac{bh^3}{12}. \tag{2}$$

For the static Euler–Bernoulli beam, the transverse angular momentum balance equation is [13]

$$\frac{d^2M}{dx_3^2} = q. \tag{3}$$

Consider now a beam in which the axial force P acts. The bending moment about the origin, $\mathbf{M}^*$, is

$$\begin{aligned}\mathbf{M}^* &= M\mathbf{j} + w(x_3)\mathbf{i} \times (-P\mathbf{k})\\ &= (M + w(x_3)P)\mathbf{j}\end{aligned} \tag{4}$$

where M is given by (1). Hence

$$\mathbf{M}^* = M + w(x_3)P. \tag{5}$$

Replacing M in (3) by $\mathbf{M}^*$ we obtain

$$\frac{d^2M}{dx_3^2} + P\frac{d^2w}{dx_3^2} = q. \tag{6}$$

Using (1) for M, (6) becomes

$$EI\frac{d^4w}{dx_3^4} + P\frac{d^2w}{dx_3^2} = q, \tag{7}$$

which is the required ordinary differential equation for $w(x_3)$. Eq. (7) agrees with the beam equation derived by Segel and Handelman [13].

We assume that the ends of the beam are clamped by the pillars at $x_3 = 0$ and $x_3 = L$ so that (7) should be considered with the boundary conditions

$$\begin{aligned} &w(0) = 0, \quad \frac{dw}{dx_3}(0) = 0;\\ &w(L) = 0, \quad \frac{dw}{dx_3}(L) = 0. \end{aligned} \tag{8}$$

Next, we nondimensionalise the problem (7) and (8). We start by considering nondimensional variables $\overline{x_3}$ and $\overline{w}$ where

$$\overline{x}_3 = \frac{x_3}{L}, \quad \overline{w} = \frac{w}{S} \tag{9}$$