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Incorporating scale effect into a multiaxial failure criterion for intact rock

Hossein Masoumi *, Serkan Saydam, Paul C. Hagan

School of Mining Engineering, UNSW Australia, Kensington NSW 2052, Australia

A R T I C L E I N F O

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A B S T R A C T

The design of structures on or within a rock mass requires an estimation of the strength of the intact rock blocks. These blocks can be many orders of magnitude greater in scale than the core samples typically tested. Thus, including the scale effect into a failure criterion is required for a more realistic estimation of rock strength with various scales. In this paper, a modified multiaxial failure criterion with scale effect parameters was developed based on the model proposed by Christensen. The modification process was carried out according to the proposed scale effect equations for different stress paths, such as uniaxial compression, point loading, uniaxial tension and pure shearing. Furthermore, the scale dependency of the proposed parameters for the modified failure criterion was assessed using the results from the uniaxial compressive and point load tests. Finally, it was confirmed that one of the modified failure criterion parameters is scale independent when the results from uniaxial compressive and point load tests are included.

1. Introduction

The scale effect is a significant characteristic in brittle and quasi-brittle media such as rock. Many studies have explored the scale effect with regards to the uniaxial compressive test in different rock types.[1–15] Some research has also investigated the scale effect for different stress paths such as point load and tensile tests.[16–26] On the other hand, investigations into the mechanical behaviour of intact rock has resulted in various failure criteria.[27–33] Some criteria [31,32,34–38] are extensions of those developed for soils, which scale effect has not been incorporated. Perhaps the most widely used criterion is that of Hoek and Brown,[39] and scale effect has been incorporated into it by assigning a scale dependence to the uniaxial compressive strength term which appears in its definition. However, there was no clear analytical justification for this approach to incorporate scale effect, and its suitability to capture scale effect for rocks brought to failure in paths other than uniaxial compression such as point loading, uniaxial tension and pure shearing remains unverified.

An alternate multiaxial failure criterion that incorporates scale effect is presented here. It is an extension of the simple two parameter multiaxial failure criterion for brittle materials proposed by Christensen[1] which is modified to include scale effect. With this modification the scale effect under different stress paths such as uniaxial compression, point loading, uniaxial tension and pure shearing can be suitably captured. Finally, a parametric study is carried out in order to investigate the scale dependency of the proposed parameters for the modified multiaxial failure criterion.

2. Background

The original criterion of Christensen,[1] with no account of scale, will be used as a basis in this study. It states that a material is not at failure when

$$\frac{\chi\kappa}{\sqrt{3}}I_1 + (1+\chi)^2\left(\frac{I_1^2}{3} - I_2\right) < \frac{\kappa^2}{1+\chi} \quad \text{or}$$
$$\frac{\chi\kappa}{\sqrt{3}}I_1 + (1+\chi)^2 J_2 < \frac{\kappa^2}{1+\chi} \tag{1}$$

where I_1 and I_2 are the first and second invariants of the stress tensor and $J_2 = (I_1^2/3) - I_2$ is the second invariant of the deviatoric stress tensor, with tensile stresses being taken as positive. Writing the stress tensor as:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{2}$$

leads to the definitions:

$$I_1 = \sigma_x + \sigma_y + \sigma_z \quad \text{and} \quad I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}{}^2 - \tau_{xz}{}^2 - \tau_{yz}{}^2 \tag{3}$$

* Corresponding author.
E-mail address: hossein.masoumi@unsw.edu.au (H. Masoumi).

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For uniaxial compression, where σ^c is the applied uniaxial compressive strength, $I_1 = \sigma^c$, $I_2 = 0$ and $J_2 = (\sigma^c)^2/3$. It is clear from the criterion that for a particular material, two parameters χ and κ are required to characterize its strength. The parameter χ is a dimensionless shape parameter and represents the ratio between the characteristic uniaxial compressive and tensile strengths, $\sigma^c$and σ^t respectively, through

$$\chi = \frac{|\sigma^c|}{\sigma^t} - 1 \tag{4}$$

The parameter κ is a strength parameter giving the criterion, the dimensions of stress and is defined as

$$\kappa = \frac{1+\chi}{\sqrt{3}}|\sigma^c| \tag{5}$$

Uniaxial compressive and tensile tests would be sufficient to evaluate χ and κ, although as will be demonstrated later, other combinations of tests involving different stress paths would suffice.

If $\chi = 0$, then the material behavior at failure is of the von Mises type as the criterion in Eq. (1) simplifies to $J_2 < \kappa^2$ and κ individually controls failure. When $\chi = 0$, the criterion defines a failure surface in the three-dimensional stress state as a right circular cylinder, symmetrical about the hydrostatic axis with radius $\sqrt{2}\kappa$. If $\chi > 0$ then Eq. (1) defines a failure surface in the three-dimensional stress space that is a revolved paraboloid symmetric about the hydrostatic axis. When $\chi \to \infty$ the material is unable to sustain load of any kind and disintegration occurs. Christensen [1] suggested that κ is a measure of the strength of the material having no micro structural damage and must be related to atomic scale properties. Furthermore, Christensen[1] suggested that χ represents the effects of micro structural deviations from the ideal sample with no micro damage. However, the attribution of micro structural origins to κ and χ has not actually been supported by targeted experimental investigations, although it is aimed in this research to provide some support to Christensen's ideas using the data gathered below.

Goodman[40] showed compressive and tensile strengths for a range of rock types indicating that, if Eqs. (4) and (5) apply, $1+\chi$ varies from about 10 to 170. It is therefore sensible to assume for all rock types that $\chi > 9$.

The challenge is to define κ and χ as functions of scale, as the criterion in Eq. (1) would then become scale dependent. The reason that only these two constants should be scale dependent is that they describe the material characteristics while the other parameters such as I_1 and J_2 are stress-dependent. For example in a rock sample with the same material characteristics, I_1 and J_2 can attain different values under various stress conditions.

3. Incorporating scale and Weibull statistics into strength measurements

The Weibull [41] probability distribution function was proposed to describe the survival probability of a block of volume V contained within a larger volume of material V_r. Bazant et al. [42] described the mathematical principles of the statistical model in a simple and elegant way. According to Bazant et al.,[42] in a chain, if the failure probability of an element (link) is assumed P_1 then the chance of survival would be ($1-P_1$) and therefore, in the case of many connecting elements, the survival probability would be as follows:

$$(1-P_1)(1-P_1)(1-P_1)(1-P_1)\ldots(1-P_1)or(1-P_1)^N = 1-P_f \tag{6}$$

where P_f is the failure probability of the chain. So,

$$N\ \ln(1-P_1) = \ln(1-P_f) \tag{7}$$

In practice, P_1 has a very small value. This leads to $\ln(1-P_1) \approx -P_1$. Therefore,

$$\ln(1-P_f) = -NP_1 \tag{8}$$

Now, by setting $N = V/V_r$, Eq. (6) becomes

$$\ln(1-P_f) = -(V/V_r)P_1 \text{ or } P_f(\sigma) = 1 - \exp\left[-\frac{V}{V_r}P_1(\sigma)\right] \tag{9}$$

where V is the volume of the sample, V_r represents the volume of one element in the sample, $P_f(\sigma)$ is the material strength, and $P_1(\sigma)$ is the strength of the representative sample. Equation (9) is the initial statistical model proposed by Weibull [43]; later, he introduced a more general form of Eq. (9) through:

$$m\ \log\left(\frac{P_f(\sigma)}{P_1(\sigma)}\right) = \log\left(\frac{V}{V_r}\right) \tag{10}$$

where m is a material constant introduced for better simulation of the size effect behaviour ($m=1$ was assumed in Eq. (9)). In Eq. (10), V and V_r can be substituted by any characteristic measure of volume such as length^3 or sample diameter^3. For example, in the case of cylindrical samples with identical shape and constant length to diameter ratio instead of volume the diameter can be substituted.

It will now be demonstrated that Eq. (10) can be applied to strengths observed in rock when subjected to different stress paths.

3.1. Scale effect in uniaxial compressive strength

Uniaxial compressive strength (UCS) of rock samples measured in a laboratory is well-known to be scale dependent and obeys Eq. (10) written in a slightly different form:

$$\frac{\sigma^c}{\sigma^c{}_{50}} = \left(\frac{d}{50}\right)^{-k_1} = \beta_1 \tag{11}$$

where the measured UCS (σ^c) is a function of sample diameter d (in millimeters) and $\sigma^c{}_{50}$ is the characteristic UCS measured of a sample with a diameter of 50 mm. Hoek and Brown[15] collected UCS results from different rock types and suggested that the value of k_1 is 0.18 (see Fig. 1).

For a uniaxial compressive test:

$$\sigma^c = -\frac{\sqrt{3}\kappa}{\chi+1} \cdot \text{and} \cdot \sigma^c{}_{50} = -\frac{\sqrt{3}\kappa_{50}}{\chi_{50}+1} \tag{12}$$

where κ_{50} and χ_{50} are material properties for 50 mm diameter samples, and κ and χ are material properties for samples of diameter d. An expression linking κ_{50}, χ_{50}, κ and χ to β_1 (or $d/50$) is then obtained by substituting Eq. (12) into Eq. (11):

$$\frac{\kappa(\chi_{50}+1)}{\kappa_{50}(\chi+1)} = \left(\frac{d}{50}\right)^{-k_1} = \beta_1 \tag{13}$$

3.2. Scale effect in point load strength index

A scale effect is also observed in strength measured using point load test (PLT). Franklin[44] showed that the point load strength index $I_s = f/d^2$, where f is the force required to fail a sample of characteristic diameter, according to:

$$\frac{I_s}{I_{s50}} = \left(\frac{d}{50}\right)^{-k_2} = \beta_2 \tag{14}$$