



Constitutive modelling of compaction localisation in porous sandstones



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ABSTRACT

Porous sandstones exhibit localised modes of failure in the form of shear or compaction bands. In particular, shear localisation is associated with brittle behaviour under low confining pressures, while under increasing confining pressures, their failure gradually becomes more ductile with the onset and propagation of compaction bands. The orientation and size of these localisation bands vary with the loading and material properties. As a consequence, a correct description of the behaviour of porous sandstones must take into account all these characteristics of localised failure. As a departure from existing approaches that take into account the orientation of localisation band, we present in this study a new approach to incorporate both size and orientation of localisation zone in constitutive models for geomaterials. Based on this approach, a breakage mechanics constitutive model is enhanced with localised mode of failure and used for the post-localisation analysis of porous sandstones. The concepts together with technical details, and numerical results are presented to show the potentials of the new approach.

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1. Introduction

In advanced engineering analysis, constitutive models play a key role in correctly predicting the load carrying capacity and failure of materials and structures. In particular, they are the tools to bring understanding gained from experiments to practical predictions of failure, towards better and safer designs. Constitutive modelling in this sense gives a mathematical description of material behaviour under different loading conditions. It is therefore essential that this description follows closely the observed behaviour of the materials in experiments and/or real life. This requires not only advanced experimental techniques, but also a generic and strong theoretical framework to transfer this understanding to a constitutive model.

In relation to porous sandstones and their mechanical behaviour, experimental techniques have witnessed many quick advances in the last few decades, with sophisticated techniques such as X-Ray and Digital Image Correlation^{1–7} for the observation of failure initiation at the micro scale. They give a deep insight into the failure mechanism of the material and the links with the evolution of its microstructures. In this sense, grain crushing and pore collapse, the two key characteristics of compaction failure in porous sandstones, have been addressed and observed extensively

at both macro and grain scales.^{5–14} In particular, the oscillation of stress–strain curves observed in triaxial experiments on sandstones under high confining pressures¹³ have been linked with the propagation of compaction bands in the specimens during loading. The sandstone specimen in such cases is gradually populated with several bands, one after another, towards the end of the failure process and the occurrence of a compaction band is associated with a cycle of stress drop and increase.

Nevertheless, in our opinion, the development of theoretical frameworks to accommodate these experimental observations and associated data is still lagging behind, despite the development of advanced constitutive models, such as those based on breakage mechanics theory,^{15–20} or the adaptation of existing plasticity based ones for porous rocks.^{21–34} In particular, one of the most important features of failure in geomaterials in general and sandstones in particular – localised failure mode and the associated size effects – is still an outstanding issue. In this sense, most (if not all) existing constitutive models for porous rocks can only detect the onset of localisation, but are not able to provide a meaningful description of the post-localisation behaviour.

As addressed earlier, localisation of deformation in geomaterials takes place after a homogeneous stage of deformation (also called diffuse failure, to differentiate with localised failure). It manifests in the form of thin bands such as shear bands in soils, and/or compaction bands in porous rocks. The material outside a localisation band usually unloads elastically, giving the indication that most or all inelastic behaviour happens inside this thin

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localisation band. In addition, while the orientation of a localisation band depends on loading conditions,³⁵ the size and behaviour of the material inside this band are material characteristics. It can be seen that the use of a single strain measure for the whole volume crossed by a localisation band, as in classical continuum models, is not adequate, due to lack of details on the size and behaviour of the localisation band to guide the model prediction. A correct description of failure must therefore account for the transition from diffuse to localised mode, and the progression of material failure beyond that. This includes the onset of localised failure and orientation of localisation band, together with the evolution of the material behaviour inside this band.

In constitutive modelling, most existing models ignore localised failure and size effect issues, either partly or completely, in model developments and interpretation/mapping of experimental data. In particular, they possess no details on the size, orientation and evolution of the localisation band. As a consequence, their behaviour does not scale with the volume, and hence size effects cannot be captured. This requires additional treatments to be incorporated later, after the model development, should these models be used in the analysis of failure that involves numerical methods for the solutions of boundary value problems. Typical examples of these treatments are the nonlocal/gradient regularisation,^{36–38} Cosserat models³⁹ and viscous regularisation.^{18,40,41} Other examples in capturing localised failure and size effects involve enhancements to the discretisation techniques (e.g. finite element methods) to correctly describe the kinematics of localised failure and behaviour.^{42–49}

In this study, we start from a key characteristic of localisation in constitutive modelling of geomaterials: kinematics of localised failure, and enhancements are developed after enriching the kinematics of constitutive models. In this sense, an enriched constitutive model will possess more than one stress–strain relationships to correctly describe different material behaviour in and out-side the localisation zone. Size effects are automatically taken into account, thanks to the intrinsic material length scale in the proposed framework. A new formulation will be established and then applied to the analysis of compaction localisation in porous sandstones. It is used in conjunction with the recently developed breakage mechanics theory^{15,16} for the analysis of post-localisation behaviour at the constitutive level. This is a key feature of the proposed approach that, in our opinion, has not been available in any existing model. Numerical examples at the constitutive level are used to illustrate the key features of the new theoretical framework.

2. A constitutive model for porous sandstone based on breakage mechanics

A breakage mechanics model is used to describe the behaviour of porous sandstones. The advantages of this model are that it allows the continuous tracking of the evolving grain size distribution due to grain crushing in cemented granular materials. To the best of our knowledge, this is so far the only constitutive model capable of linking macro behaviour with an internal variable (breakage variable) representing the microstructural changes due to grain crushing in a consistent, yet simple and efficient manner. This follows the pioneering work on breakage mechanics theory and its applications by Einav.^{15,16} In this section, a brief description of a model based on breakage mechanics theory is given to set a background for the enhancement in the next section. This is then followed by the performances of the model in capturing the behaviour of sandstones under a triaxial loading.

2.1. Model description

The breakage mechanics theory for crushable granular materials¹⁵ was built on the micromechanics of grains, using statistical homogenisation to upscale the grain-scale energy potential to obtain the macro energy potential of the continuum model. A simple breakage model proposed by Einav,¹⁶ following the thermodynamic framework described in Ref. 50, and later improved by Nguyen and Einav⁵¹ is considered in this study. The following standard notations are used: bulk and shear modulus K and G , respectively; mean effective stress p and deviatoric shear stress q ; total and elastic volumetric strains ε_v and ε_v^e ; total and elastic deviatoric shear strains ε_s and ε_s^e ; total and deviatoric stress tensors σ_{ij} and s_{ij} ; and total, elastic and plastic strain tensors ε_{ij} , ε_{ij}^e and ε_{ij}^p .

The stress–strain relationship in the triaxial stress space is:

$$p=(1-\vartheta B)K\varepsilon_v^e \quad (1)$$

$$q=3(1-\vartheta B)G\varepsilon_s^e \quad (2)$$

where the grading index ϑ is a result of a statistical homogenisation,¹⁵ and can be obtained from the initial and ultimate grain size distributions (gsd). Physically ϑ is related to the crushing potential of the materials and admits values from 0 to 1. The internal variable B , representing the degree of grain crushing, is used to track the evolution of the current gsd.

Einav¹⁶ derives an elastic–plastic–breakage yield criterion to describe the frictional shear dissipation at the macro scale. This yield criterion y is written as:

$$y=\frac{(1-B)^2 E_B}{E_C}+\left(\frac{q}{Mp}\right)^2-1 \leq 0 \quad (3)$$

where M is the slope of the critical state line in the $p-q$ space; E_C is the critical breakage energy which is computed from the experimentally provided critical crushing pressure P_{cr} in isotropic loading condition as:

$$E_C=\frac{P_{cr}^2 \vartheta}{2K} \quad (4)$$

and E_B is the breakage energy, the thermodynamical conjugate to the breakage variable B , and has the following form:

$$E_B=\frac{\vartheta}{2(1-\vartheta B)^2}\left(\frac{p^2}{K}+\frac{q^2}{3G}\right) \quad (5)$$

The evolution laws for breakage and plastic strains are derived from the yield function written in mixed energy–stress space⁵¹ and denoted as y^* . Simply speaking, y^* acts as a plastic potential in classical plasticity theory and provides the model with evolution rules for breakage and plastic strains.

Since y^* is different from the yield function y , the model intrinsically possesses non-associated flow rules (Fig. 1). For the sake of simplicity, we skip the details on y^* , which can be found in Ref. 51, and just present the flow rules that are necessary for the implementation of the model:

$$dB=d\lambda \frac{\partial y^*}{\partial E_B}=d\lambda \frac{2(1-B)^2 E_B \cos^2 \omega}{E_C} \quad (6)$$

$$d\varepsilon_v^p=d\lambda \frac{\partial y^*}{\partial p}=d\lambda \frac{2(1-B)^2 E_B \sin^2 \omega}{pE_C} \quad (7)$$

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