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Two-dimensional displacement discontinuity method for transversely isotropic materials

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ABSTRACT

This paper presents the fundamental solution for a two-dimensional displacement discontinuity method (DDM) for transversely isotropic elastic materials. We follow the procedures shown in the literature, in which there are some typographic errors and a lack of proper explanations for some expressions. Based on the fundamental solution of deformation due to a single point force in transversely isotropic materials, the formulation for deformation from force dipoles has been revisited. Generalised Hooke's Law is used to establish the relationship between dipole strengths and displacement discontinuities, which leads to the fundamental formulation of DDM for transversely isotropic materials. We present the full details of derivation and corrections to some expressions which have previously been presented with errors. In addition, we present the fundamental solution expressions for DDM for one situation which was not included in the literature. The fundamental solutions are implemented in an existing DDM code, FRACOD, and the method is verified by some examples with an analytic solution and finite element method. Furthermore an engineering application is simulated with the scheme.

1. Introduction

Rocks are often modelled as isotropic material. But in some situations, such as for some sedimentary rocks,[1] such models are not accurate. Sedimentary rocks often contain numerous bedding structures and exhibit strong mechanical anisotropic behaviour. In simulations of sedimentary rocks, anisotropic properties should be considered. Transverse isotropy is a special anisotropic material property in which material possess an axis of symmetry of rotation and the material has isotropic properties in a plane normal to that axis.[2] Five independent material parameters are needed to describe the general constitutive relations for transversely isotropic material, and four parameters for in-plane deformations in the plane containing the transverse direction. Therefore the four parameters for plane deformations can be derived from the five of the general constitutive relations.

The expressions for the general solution for displacements and stresses in transversely isotropic elastic body in two-dimensional problems have been obtained with Lekhnitskii,[2] Stroh[3] or other methods; see Ref. 4. Lekhnitskii[2] used stress functions and compatibility equations while Stroh[3] sought a solution for the equilibrium equations in terms of displacements. The solutions are expressed in complex variables. They depend on three pairs of complex conjugate characteristic roots of a sixth order characteristic equation for general plane problems which contain out-of-plane shear, and two pairs for usual completely plane problems without out-of-plane shear. The characteristic values are purely dependent on material parameters. The general expressions have been used for various problems, particularly the solution for displacements and stresses in infinite body caused by a concentrated force. This concentrated force solution can be used as the basis for some other problems.

Displacement discontinuity method (DDM) for isotropic materials has been used successfully in many engineering simulations, particularly for simulation of crack growth; see Crunch and Starfield[5] and Shen[6] for two-dimensional cases and Shi et al.[7] for three-dimensional problems. But it has not been employed widely for anisotropic materials. DDM, as a boundary element method

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(BEM) has an advantage over other domain discretisation methods (such as the popular finite element method (FEM)) of formulating problems with one dimension less than the problem's real dimensions. For crack propagation problems, it also has an advantage over some other BEMs by only discretising the cracks themselves, not the two surfaces of the cracks. These reduce the total number of equations to be solved. However, it requires the analytic expression of fundamental solution, which is only available for some problems.

Kayupov et al.[8] and Kimence and Erguven[9] are among those who formulate and use DDM for transversely isotropic materials, and they follow a similar procedure. Kayupov et al.[8] first used the concentrated force solution to obtain solutions for displacements and stresses caused by a force dipole and then used the relations between force dipoles and strains to achieve the expressions of solutions for displacements and stresses in two-dimensional anisotropic material in terms of displacement discontinuities. The source of these relations was not given. They are in fact the generalised Hooke's law since the force dipoles are related to stress components. This forms the basis of DDM for anisotropic materials in two-dimension with complex variables. However there are no examples given in Ref. 8. Denda[10] presented the fundamental solution due to a dislocation dipole in an infinite body, which can also be used for DDM.

Most recently, Bobet and Garcia Marin[11] employed this for completely plane problems of transversely isotropic materials. However they believed that the complex characteristic roots for this type of materials must be in pure imaginary form (discussed in Bobet[12]) and obtained the corresponding expressions for displacements and stresses in real variables. We believe that his argument in Ref. 12 for the pure imaginary characteristic roots is inaccurate, given that he discussed only one of two possible cases (see later discussion). While for some materials the complex characteristic roots are indeed pure imaginary, there are materials which produce full complex characteristic roots. In this paper, we show the fundamental solution for DDM for transversely isotropic materials which have full complex characteristic roots and implement it in an existing fracture simulation code FRACOD.

Firstly in Section 2, we summarise Kayupov's[8] procedure of deriving the fundamental DDM solution for transversely isotropic materials. He listed a number of equations without proper explanations. We will show the sources and detailed derivation for these equations. In Section 3 we will discuss the inaccuracy of Bobet's[12] conclusion about the form of the characteristic roots, subsequently presenting (for purpose of completeness and correcting some of the typographic errors in their expressions) the expressions for DDM for the cases in which the characteristic roots are purely imaginary. The fundamental DDM solutions for transversely isotropic materials with the full complex conjugate characteristic roots are given in Section 4. Brief description of DDM equations and some examples for validation with analytic solution and finite element method will be given in Section 5 before conclusion in Section 6.

2. Fundamental DDM solution for transversely isotropic materials

We consider transversely isotropic materials whose isotropic symmetry is orthogonal to the y-axis of a global coordinate system i.e. the materials are isotropic in planes normal to the y-axis. We assume that the deformation is under plane strain conditions in planes parallel to xy coordinate plane i.e. $\varepsilon_z = 0$. The constitutive relations for strains and stresses under the plane strain condition are expressed[11,12] as follows:

$$\begin{aligned} \varepsilon_x &= \beta_{11}\sigma_x + \beta_{12}\sigma_y, \\ \varepsilon_y &= \beta_{12}\sigma_x + \beta_{22}\sigma_y, \\ \gamma_{xy} &= \beta_{66}\sigma_{xy}, \end{aligned} \tag{1}$$

where the compliance coefficients $\beta_{ij}(i, j = 1, 2)$ are related to Young's moduli, Poisson's ratios and shear modulus in the material symmetry directions according to

$$\beta_{11} = \frac{1-\nu_{xz}^2}{E_x}, \beta_{12} = -\frac{1+\nu_{xz}}{E_y}\nu_{yx}, \beta_{22} = \left(1 - \frac{E_x}{E_y}\nu_{yx}^2\right)\frac{1}{E_y}, \beta_{66} = \frac{1}{G_{xy}}. \tag{2}$$

The material properties are different in the x and y directions and are isotropic in xz plane; E_x and E_y are the Young's moduli in the x and y directions; G_{xy} is the shear modulus in the xy plane; ν_{xz} is the isotropic Poisson's ratio in the xz plane; ν_{yx} is the Poisson's ratio in the xy plane and represents the shrinkage in the x-direction caused by tension in the y-direction. It is noted that the Poisson's ratio in the xy-plane representing the shrinkage in the y-direction by tension in the x-direction ν_{xy} satisfies the relation $E_y\nu_{xy} = E_x\nu_{yx}$.

The solution for plane strain deformations of transversely isotropic material can be obtained by using Lekhnitskii's[2] or Stroh's[3] methods for generalised or complete plane strain deformations[8] of general anisotropic materials. The generalised plane strain deformations are independent with one spatial coordinate but can have shear strains in this coordinate direction. When the materials are transversely isotropic, some of the material parameters of the general anisotropic materials become zero. If deformation further satisfies the standard plane strain conditions in xy-plane (no shear strain components in the z-direction), then the displacements and stresses can be expressed with two analytic complex functions. The exact form of the two analytic complex functions depends on the considered problems. For example, as shown later, the solution of a single concentrated force has logarithmic form, while the solution of force dipole is inversely proportional to the distance from the centre of the force dipole to the point where the solution is evaluated.

2.1. Deformation from a single concentrated force

The displacements and stresses at point (x,y) in an infinite transversely isotropic body due to a concentrated force (f_x, f_y) at point (x^*,y^*) are given by[8].

$$\begin{aligned} u_x &= 2\,\mathrm{Re}\left\{p_1A_1\ln\left(\xi_1-\xi_1^*\right) + p_2A_2\ln\left(\xi_2-\xi_2^*\right)\right\}, \\ u_y &= 2\,\mathrm{Re}\left\{q_1A_1\ln\left(\xi_1-\xi_1^*\right) + q_2A_2\ln\left(\xi_2-\xi_2^*\right)\right\}, \\ \sigma_{xx} &= 2\,Re\left\{\mu_1^2\frac{A_1}{\xi_1-\xi_1^*} + \mu_2^2\frac{A_2}{\xi_2-\xi_2^*}\right\}, \\ \sigma_{yy} &= 2\,Re\left\{\frac{A_1}{\xi_1-\xi_1^*} + \frac{A_2}{\xi_2-\xi_2^*}\right\}, \\ \sigma_{xy} &= -2\,Re\left\{\mu_1\frac{A_1}{\xi_1-\xi_1^*} + \mu_2\frac{A_2}{\xi_2-\xi_2^*}\right\}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} &\xi_i = x + \mu_i y, \qquad \xi_i^* = x^* + \mu_i y^*, \\ &p_i = \beta_{11}\mu_i^2 + \beta_{12}, \quad q_i = (\beta_{12}\mu_i^2 + \beta_{22})/\mu_i \quad (i = 1, 2) \end{aligned} \tag{4}$$

$\mu_1$andμ_2 are two distinct roots (characteristic values) of the characteristic equation