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Effect of material anisotropy on time-dependent wellbore stability

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ABSTRACT

The paper presents a numerical wellbore stability model using the finite element method (FEM). The core driver of the model uses Biot's theory of poroelasticity. The model uses a time-dependent analysis that accounts for the poroelastic effects of pore pressure and deviatoric stress, which are not accounted for in the common static linear elasticity analyses. The use of pseudo-3D FEM based on the theory of generalized plane strain, allows the freedom to analyze complex geometries such as inclined boreholes in anisotropic formation under three dimensional stress field. Shear failure prediction is demonstrated using isotropic and anisotropic failure criteria. Available analytical solutions in literature for anisotropic poroelasticity assume transversely isotropic material with the plane of isotropy always perpendicular to the borehole axis. The presented numerical model removes the above assumption and accounts for the mechanical anisotropy and formation dipping effects that are difficult to deal with using an analytical-based analysis. Finally, juxtaposition between isotropic and anisotropic analyses shows significantly dissimilar failure prediction and mud weight recommendations.

1. Introduction

The theory of consolidation of saturated porous media was originally developed by Biot[1] and widely referred to nowadays as the theory of poroelasticity. It describes the rock behavior under three-dimensional stress field coupled with the hydraulic effect due to the presence of saturating fluid inside the pores. Using Biot's theory, Detournay and Cheng[2] developed an analytical solution for vertical boreholes in isotropic formations under internal and external two-dimensional stress field with the plane strain assumption. This time-dependent solution has been widely used in the oil and gas industry to analyze borehole stability problems that can be detrimental to drilling operations. Cui and Abousleiman[3] extended Detournay and Cheng's work to include inclined boreholes in isotropic formations under three-dimensional stress field using the generalized plane strain theory developed by Amadei and Leknitskii.[4,5]

Rather than assuming an isotropic formation, Abousleiman and Cui[6] incorporated a special case of material anisotropy into the formulation known as transverse isotropy (TI) that is common in shale formations, where the material properties vary only in two orthogonal planes. Due to its laminated nature, shale formations are highly anisotropic. Assuming isotropic rock models will result in erroneous predictions of wellbore shear and tensile failures. However, they assume that the plane of isotropy of the TI material is always perpendicular to the borehole axis. This substantial assumption is made for mathematical convenience. Otherwise, the solution formulae would be lengthy and cumbersome. Though convenient, this assumption excludes the majority of the cases encountered, especially in inclined wellbores. Using numerical models to solve such complex geometries is usually recommended.

Using static wellbore stability models based on linear elasticity are not sufficient, as they do not consider the dynamic nature of the hydraulic effect of fluid flow and its effect on effective stresses. A time-dependent solution is needed using the theory of poroelasticity to monitor the integrity of the wellbore over time during and after drilling.

The topic of wellbore stability has been extensively discussed in literature. Gholami et al.[7] discuss the application of different failure criteria in the estimation of optimum mud window for drilling a safe borehole. They demonstrated the use of a well logs in estimation the mechanical properties and in-situ stresses. This work falls short as the analysis only discusses vertical boreholes drilled in isotropic formation, and no failure along weak bedding plane. The majority of subsurface rocks exhibit TI behavior.[8] Tien and Kuo[9] discuss the failure of TI rocks is governed by failure of the rock matrix and sliding along the discontinuity of weak bedding planes. Matrix failure coupled with slip failure effect on optimum drilling mud weight for inclined boreholes is discussed in literature,[10,11] but isotropic stress models are used and no time-

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dependent analysis is discussed. Observed anisotropic shear failure around the borehole caused by matrix and slip failures and their effect on estimating in-situ stresses is analyzed by Lee et al.[12] However, the authors do not consider anisotropy in mechanical properties, and a homogenous isotropic rock is assumed throughout. Yan et al.[13] address this shortcoming by analyzing borehole stability using an anisotropic linear elasticity model while accounting for rock strength anisotropy, but fall short to account for the time-dependent poroelastic effect. The time-dependent effects of thermal and chemical gradients on wellbore stability is out of the scope of this work, and has been addressed by several authors,[14–16]

This work describes the development of a numerical model for fully coupled anisotropic poroelastic model that can provide a solution for the transient state of pore pressure and effective stress. While accounting for the mechanical anisotropy, time-dependent analysis of matrix and slip failures is discussed. Finally, estimation of borehole collapse and fracture initiation pressures are analyzed for different borehole inclinations using isotropic and anisotropic models.

2. Poroelastic model

The classical plane strain assumption is a two dimensional (2D) approach, where the strain normal to the x–y plane, ϵ_z, and the anti-plane shear strains, γ_{xz} and γ_{yz}, vanish to zero. This assumption is a valid one only when the following conditions are met: (a) The borehole axis is parallel to the principal stress direction, where the z-component far-field stresses, τ_{xz} and τ_{yz}, vanish to zero, and the problem becomes a two dimensional one and (b) the material has a maximum of one plane of elastic symmetry that is perpendicular to the hole axis. This indicates that the state of stress at any point can be specified by σ_x, σ_y and τ_{xy} only. In practical terms, the assumption limits the analysis to: vertical or horizontal wells in isotropic formations, and vertical wells in transversely isotropic formations, where the plane bedding is orthogonal to the hole axis.

Due to the limitations of the classical plane strain assumption, the generalized plane strain (GPS) assumption developed by Amadei and Leknitskii[4,5] is used throughout this paper. The GPS assumption deconstructs the problem into three parts: I. Plane strain problem that involves σ_x, σ_y, σ_z, τ_{xy} and pore pressure, II. Uniaxial-strain problem that involves σ_z only and III. Anti-plane shear problem that involves τ_{xz} and τ_{yz}. Problem I can be separated into three different loading modes: 1. The effect of the mean isotropic stress, 2. The poroelastic effect of pore pressure and 3. The poroelastic effect of deviatoric stress. A detailed analytical solution for isotropic poroelasticity using the GPS assumption is developed by Abousleiman and Cui.[6]

2.1. Governing equations

The constitutive equations for isothermal poroelasticity are governed by the stress–strain relation and the variation of fluid content per unit referential volume (ζ). They are given by[6]:

$$\sigma_{ij} = D_{ijkl}\epsilon_{kl} - \alpha_{ij} p \tag{1}$$

$$\zeta = \alpha_{ij}\epsilon_{ij} + \frac{1}{M}p \tag{2}$$

where σ and ϵ are the total stress and strain vectors, D is the drained elastic stiffness tensor, α Biot's effective stress coefficient vector, M is Biot's modulus and p is the pore pressure. A repeated subscript refers to the Einstein summation. The strain can be related to the solid displacement (u) as follows:

$$\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where a comma followed by a subscript marks the spatial differentiation. Fluid diffusion is governed by Darcy's law as follows:

$$J_f = -\bar{\rho}_f[\kappa\nabla p] \tag{4}$$

$$\kappa_{ij} = \frac{k_{ij}}{\mu} \tag{5}$$

where J_f is the fluid flux, $\bar{\rho}_f$ is the average fluid density, κ is the mobility tensor, k is the rock permeability tensor and μ is the fluid viscosity. An isotropic permeability tensor is assumed throughout the paper, but the numerical formulation allows for the use of an anisotropic tensor. The conservation laws enforced on the model ignore all body forces such as gravity, and inertial forces. The mechanical equilibrium equation is given by:

$$\sigma_{ij,j}=0 \tag{6}$$

The microscopic mass balance for the system, assuming no loss of mass from the solid phase, is given by:

$$\left(\frac{\partial\zeta}{\partial t}\right) + \nabla J_f = 0 \tag{7}$$

The presented constitutive and transport equations are substituted into the conservation laws to yield the final field equations. The Navier-type equation for displacement in the x–y plane using the generalized plane strain assumption is obtained by invoking Eq. (1) into the equilibrium equation:

$$\frac{1}{2}\left[(D_{11} - D_{12})u_{i,jj} + (D_{11} + D_{12})u_{j,ji}\right] - \alpha\nabla p = 0 \tag{8}$$

Similarly, Eq. (2) is substituted into the mass balance equation to yield the local continuity equation:

$$\alpha(\nabla\bullet\dot{u}) + \frac{1}{M}\dot{p} - \kappa\nabla^2 p = 0 \tag{9}$$

where the overdot denotes the time derivative.

3. Material anisotropy

For generality, this section assumes orthorhombic (orthotropic) material, where the material properties change in all three principal directions. The stress–strain relationship is given by the generalized Hooke's law using the engineering notation for strain:

$$\begin{bmatrix}\sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{yz} & \tau_{xz}\end{bmatrix}^T = D\begin{bmatrix}\epsilon_x & \epsilon_y & \epsilon_z & \gamma_{xy} & \gamma_{yz} & \gamma_{xz}\end{bmatrix}^T \tag{10}$$

The drained elastic stiffness tensor, which is equivalent to the inverse of the compliance tensor, C, is given in the intrinsic rock properties coordinates[4,5]:

$$D = C^{-1} = \begin{bmatrix} \frac{1}{E_x} & \frac{-\nu_{yx}}{E_y} & \frac{-\nu_{zx}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{yx}}{E_y} & \frac{1}{E_y} & \frac{-\nu_{zy}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{zx}}{E_z} & \frac{-\nu_{zy}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} \end{bmatrix}^{-1} \tag{11}$$

In the above relation, E_i, ν_{ij} and G_{ij} (i, $j = x$, y and z are the