Technical note

Stochastic analysis of strength and deformability of fractured rocks using multi-fracture system realizations

Majid Noorian-Bidgoli\textsuperscript{a,b,*}, Lanru Jing\textsuperscript{a}

\textsuperscript{a} Division of Land and Water Resources Engineering (LWR), Royal Institute of Technology (KTH), Stockholm, Sweden

\textsuperscript{b} Division of Mining Engineering, Engineering Department, University of Kashan, Kashan, Iran

\textbf{A R T I C L E  I N F O}

\textbf{Article history:}
Received 5 September 2014
Received in revised form 19 April 2015
Accepted 29 May 2015
Available online 18 June 2015

\textbf{Keywords:}
Discrete element methods (DEM-DFN)
Stress–deformation analysis
UDEC
Failure Criteria
Numerical experiment
Stochastic realization

1. Introduction

It is generally recognized that the existence of unknown discontinuities with varying sizes, locations and orientations lead to a significant level of uncertainty in the physical behavior of fractured rock masses. Uncertainty and variability are caused by the nature and unpredictable behaviors of the fractured rock mass as an inherently heterogeneous geological material. Therefore, the variability of the fractured rock properties is an important and inescapable issue, which remains one of the most difficult challenges. In this regard, stochastic analysis of strength and deformability parameters of fractured rocks is a key issue of site characterization of rock engineering projects, which may provide more reliable estimation of variability of strength and deformability of the fractured rock masses concerned, as a means of quantitative representation of uncertainty/variability of the host rock mass concerned, and therefore enhancing design of surface and subsurface structures in and on rock masses, and reliable economic evaluation.

Although a number of laboratory experimental study related to strength and deformability characteristics of rocks \cite{1-3} have been conducted on the physical models or artificial rock-like materials, but it should be noted that it is difficult and in some cases impossible to estimate experimentally the variability of strength and deformability parameters of fractured rocks due to the scale effects of the samples and economic challenge for a large number of tests of different sample volumes. Also, it is not an easy task to investigate stochastically the large-scale in-situ tests.

On the other hand, the available empirical methods \cite{4-10} usually require just a single value for each input characteristic and give a single output value for mechanical properties of rock masses. They often give too conservative evaluates for property characterizations, due to the fact that they make use of categorized parameters based on case histories without a proper mathematical platform. Thus, since the difficulty of handling the inherent uncertainties of the parameter values in the empirical methods, these approaches cannot provide the probability distributions of the investigated fractured rock mass properties.

There are theoretical studies in the literature to develop constitutive models for describing the deformation behavior of rock masses \cite{11}, and to estimate the strength and deformation properties of rock masses \cite{12-14}, which have a limited applicability for variation or uncertainty evaluation requirements. Most of them used assumptions and simplification for representing fractured rock masses with much simplified or regular fracture system geometry so that the models cannot make a good representation of the complex fractured rocks in nature.

\*Corresponding author at: Division of Mining Engineering, Engineering Department, University of Kashan, Kashan, Iran.

E-mail addresses: mnoorian@kth.se, majidnoorianb@gmail.com (M. Noorian-Bidgoli).

http://dx.doi.org/10.1016/j.ijrmms.2015.05.010
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Therefore, proper numerical modeling approach becomes a feasible tool for conducting stochastic analysis of the mechanical behavior of the fractured rocks, due to its flexibility in fracture system representation and initial/boundary conditions.

Numerical methods, especially the finite element method (FEM) as the continuous or implicit methods, and discrete element method (DEM) as the discontinuous or explicit methods, have been used to assess the strength and deformation behavior of fractured rock [15–17], but not based on a systematic methodology of stochastic analysis.

There are a limited number of numerical studies [18–24] that applied stochastic analysis for evaluating the strength and deformability characteristics of rocks. However, since these stochastic analyses were performed on the data obtained from laboratory tests on the rock samples, not on the scale of REV (representative elementary volume) or engineering problems, so that the results may not be applicable for evaluating uncertainty or variability of the properties/parameters concerned, since the random nature and the stochastic distributions of the fracture system geometry of the fractured rocks were not properly handled at a proper scale.

The main objective of this study is to develop a systematic numerical platform for statistical predictions of strength and deformability parameter distributions of the fractured rocks by the numerical stochastic analysis, which incorporates the uncertainties of the intact rock and fracture parameters, and was rarely reported in literature. The work described in this paper is a part of a systematic research program to study the strength and deformability of fractured rocks at a fundamental level, as developed recently for predicting strength and deformability of fractured rocks [25–28].

In this paper, a stochastic modeling approach is developed to quantify the variations of equivalent the deformation parameters, such as Young’s modulus and Poisson’s ratio and strength parameters defined two widely-used failure criteria, namely Mohr–Coulomb (M–C) and Hoek–Brown (H–B), of fractured rock masses. The numerical models considered contain stochastically generated fracture system realizations within a suitable REV, so that the multiple realizations of discrete network fractures (DFN) captured heterogeneity in fractured rock mass properties at the REV level so that the overall behavior of the DEM models can be represented as an equivalent fractured medium of the site considered. The multiple fracture system realizations are created by identifying geometric parameters of joint sets and defining the distribution forms of fracture location, orientation, and length using Monte Carlo simulations. To continue, the sets of so-called “numerical experiment” are adopted, in the similar ways for the standard uniaxial and biaxial laboratory testing on intact rock samples, to generate stress–deformation behaviors of all stochastic DEM models for probabilistic analysis the variation of strength and deformability of fractured rocks concerned. The code applied is the DEM code UDEC [29], since it can represent explicitly the fracture system geometry and constitutive models of rock matrix and fractures, which cannot be handled by continuum modeling methods. The problem was defined in a 2D space for simplicity since the problem is a generic study for reaching a fundamental understanding about stochastic evaluation of fracture rock behavior, which cannot be tested in laboratory conditions at REV volumes at present.

2. Description of numerical experiments

The methodology adopted is a systematic numerical simulating of typical laboratory compression test conditions based on procedures developed by Noorian et al. [25] to conduct a series of uniaxial and biaxial numerical experiments on a large number of the fracture network realizations using UDEC.

The numerical experiments were performed in a two-dimensional (2D) space and under quasi-static plane strain conditions without considering the effects of gravity for stress–deformation analyses.

Fractured rock was assumed a crystalline hard rock mass without considering strain-softening since only the strength and elastic deformability parameters are concerned. The rock concerned is a volcanic type at Sellafield, UK, where a comprehensive site investigation was conducted in the past, as the properties of the intact rock and fractures are shown in Table 1, which were obtained from a site investigation program of Nirex [30] and related laboratory tests.

Based on measured data from site investigations and the available constitutive models in the UDEC, the intact rock matrix was defined as a linear, isotropic, homogeneous, and elastic material, and fractures were defined to follow an ideal elasto-plastic behavior of a Mohr–Coulomb model in the shear direction and a hyperbolic behavior (Bandis’ Law) in the normal direction (Table 1). Also, the initial aperture of fractures (without stress) was assumed to be a constant of 65 µm during simulation, see Table 1.

In this paper, the cracking and crushing of rock blocks during loading processes was not considered.

2.1. DFN realizations establishment

The DEM method has ability for creating stochastic geometric models of the complex fracture systems, reflecting the uncertain and heterogeneous nature of fracture systems whose representative properties and parameters can only be partially measured or estimated. Hence, to realize the stochastic fracture system for the numerical experiment, a set of stochastic DFN models was generated.

An independent DFN generator was developed using the MATLAB workspace for generating the DFN realizations, based on the method developed by Min and Jing [31]. It is able to create the

Table 1

<table>
<thead>
<tr>
<th>Intact rock</th>
<th>Mechanical properties of intact rock and fractures used for the UDEC modeling [25].</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2500 kg/m³</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>84.6 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.24</td>
</tr>
<tr>
<td>Uniaxial compressive strength</td>
<td>157 MPa</td>
</tr>
<tr>
<td>Fracture initial normal stiffness</td>
<td>434 GPa/m</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>434 GPa/m</td>
</tr>
<tr>
<td>Friction angle</td>
<td>24.9</td>
</tr>
<tr>
<td>Cohesion</td>
<td>0°</td>
</tr>
<tr>
<td>Dilatancy</td>
<td>5°</td>
</tr>
<tr>
<td>Aperture for zero normal stress</td>
<td>65 µm</td>
</tr>
<tr>
<td>Residual aperture at high stress</td>
<td>1 µm</td>
</tr>
<tr>
<td>Shear displacement for zero dilation</td>
<td>3 µm</td>
</tr>
</tbody>
</table>

Note: Uₙ₁, Uₙ₂, Uₛ₁, Uₛ₂, eₙ₁, eₙ₂, and eₛ are parameters control fracture mechanical behavior in normal direction [32].