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Perturbation analysis for predicting the temperatures of water flowing through multiple parallel fractures in a rock mass

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ABSTRACT

In this paper, we revisit this problem of heat extraction from an enhanced geothermal system (EGS) with multiple parallel fractures, which provide more surface area and conductive paths between injection and production wells, by employing a semi-analytical method. A dimensionless formulation is obtained first to find the controlling parameters. By using the Laplace transformation, the original partial differential equations (PDEs) are converted into a series of ordinary differential equations (ODEs) that can be solved analytically. A dimensionless parameter, the Peclet number *Pe*, which reflects the rate of the heat removed by the flowing fluid in the fracture from the surrounding formation, is then used as the small parameter for the regular perturbation analysis. The approximate solutions at higher order are obtained for the rock and fluid temperatures. In addition, numerical results based on a fourth-order Runge–Kutta (R–K) method are presented and match well with these by perturbation analysis. Our solutions also match the exact solutions, which exist for specific conditions. A few leading terms in the asymptotic expansion are sufficient to provide accurate results. The results demonstrate that optimal values of the fracture number and spacing can be found to maximize the productivity and lifetime of an EGS system.

1. Introduction

As one technology to extract heat from subsurface resources, the approach embodied in enhanced geothermal systems has attracted a great deal of interest during the last several decades, commercially and theoretically. For example, EGS projects which are currently either under development or operating around the world include the Soultz project in France with rated capacity 1.5 MW at depth of 4.2 km, the Landau project in Germany with a rated capacity of 3 MW at a depth of 3.3 km and the Cooper Basin project in Australia with an a targeted capacity of 250–500 MW at a depth of 4.3 km [1].

Theoretical studies on the geothermal energy recovery associated with circulation through hydraulic or natural fractures focuses on the mathematical modelling of the heat transfer/exchange between the rock formation and the fluid [2]. One important objective is to select values for the factors that optimize performance of the system. Relevant factor include the number of fractures, fracturing spacing, and well separation. Based on the complexity of the problems studied, the approaches can be classified into the following three types. First, analytical or semi-analytical solutions can be found only for simple cases. For example, by assuming a fluid flow with an uniform velocity in a straight channel or fracture, numerous researchers found analytical solutions by using Laplace transformation or Green's functions [3–7]. Second, analytical solutions have been obtained for radial flow [8–10]. Third, the analytical solution can still be obtained for more complicated cases if some simplifications are made. For example, the fluid and heat flow in a single fracture penetrated by dipole wells was analyzed in [11–14].

One of the most important objectives for EGS designs is to improve the productivity and efficiency of the system and provide a sustainable power output. As a single fracture system has limited exposed area for heat transfer to the flowing fluid, it is less efficient than a multiple fracture system, as pointed out in [6,15]. Therefore, multiple fractures have been used as one effective approach to improve the performance of the EGS [16]. Although the natural fractures that exist with statistical distributions in the formation may improve the contact area between the fluid and the formation, they are not considered in this paper, and we instead deal with a system of parallel infinite fractures with regular spacing to find the best geometrical parameters for heat extraction, such as the number of fracture, fracture spacing and well separation.

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The transient heat transfer in a multiple-fracture (equivalently multilayered) medium have many applications in the industry, such as to air frames, nuclear reactors [17] and in geothermal engineering [16–18]. Generally, these kinds of thermal problems are solved by using a variety of approaches, for example the separation of variables [19–22], the finite difference method [23,24], finite element analysis [25,26], Green's function method [16,27], eigenfunction expansion [28,29], and the Laplace transformation method [26,30,31].

Although some of the above-mentioned multilayered models provide analytical solutions for the temperature evolution in the multilayered media, the methods used cannot simply be applied to the present problem, as explained below. As the interface is fully bonded, most of the previous models consider heat conduction in the media with temperature continuity imposed at the interface or they impose a heat flux balance condition there. In this case, the problem can be solved in a straightforward way by using Laplace transforms [30] or separation of variables [19]. However, the introduction of fluid flow in the fractures makes the problems more complicated. First, due to the difference in temperature of fluid and rock, the heat balance condition used in the above models is no longer valid. The fluid will absorb heat from or release heat to the surrounding media depending on the temperature difference. The difficulty in measuring the heat transfer coefficient (HTC) for fluid flow in fractures is well known [32–34]. Second, the fluid flow and heat front in each planar fracture move at different speeds and thus different conditions must be satisfied at each interface.

This paper considers the heat extraction from an EGS consisting of multiple parallel fractures. In Section 2, we will describe the problem studied in more detail, i.e. model description and mathematical formulation, which are followed by the construction of a dimensionless formulation in Section 3. Following [35], the fluid temperature is equal to the rock temperature at the interface, which eliminates the need to accurately measuring the HTCs. Sections 4 and 5 deal with the perturbation solution and R–K scheme, respectively. Section 6 gives the analytical solutions for the cases with single and two fractures. The method validation and computational results are given in Section 7. In the end we will compare the results obtained from the current method with those in the literature.

2. Problem formulation

The geometry for the present model is shown in Fig. 1. Initially, there exist N ($N \geq 1$) parallel rectangular fractures, which divide the half space into $N+1$ layers, and the whole system (fluid and rock formation) is in an equilibrium state with the temperature distribution $T = A_0 z + B_0$, where A_0 is the geothermal gradient, B_0 is the surface soil temperature and z is the depth. The surface temperature is fixed with a value of B_0 for all time. When time $t > 0$, a cold fluid is injected at a constant injection rate Q and constant initial temperature T_{in} from the injection points. The outlet rate at the production well is assumed to be the same as the injection rate.

The geometry of the fracture system is defined as follows: the length, width and aperture of a fracture are L, W_0 and ω^ℓ, respectively; the depth of a fracture is z^ℓ, where the fracture number ℓ is counted from bottom to top. This is to say that the bottom fracture is the first one with the vertical coordinate $z = z_0^1 = H$, and the vertical coordinate of the top fracture is z_0^N. The ground surface corresponds to $z = z_0^{N+1} = 0$. For the purpose of simplicity, the injection points are assumed to be at the origin $x = 0$ and the production points are at $x = L$.

Some assumptions are made for the present model: (1) the fluid is incompressible with Newtonian rheology and the rock is impermeable, homogenous and isotropic; (2) the apertures of all fractures are assumed to be equal and the stress-induced change of the fracture aperture is ignored; (3) the mechanical properties (density, specific heat capacity and thermal conductivity) of the rock and the fluid are constant and independent of the temperature change; and (4) the injection temperature is the same for all fractures and the injection rate for each fracture is equal to the total injection rate Q divided by the number of fractures N.

2.1. Heat transfer in the fracture

According to [7], the heat transport in the fractures contains four components, i.e. heat storage, advection, dispersion and transfer from the fracture walls, which are listed from left to right as

$$\frac{\partial T_f^\ell}{\partial t} + v^\ell \frac{\partial T_f^\ell}{\partial x} - D_L \nabla^2 T_f^\ell + \frac{\lambda_r}{\rho_w c_w \varpi^\ell} \left\{ \frac{\partial T_r^{\ell+1}}{\partial z} - \frac{\partial T_r^\ell}{\partial z} \right\} = 0 \quad at\, z = z_0^\ell, \tag{1}$$

where T_f^ℓ is the fluid temperature along the ℓth fracture and T_r^ℓ the temperature in the ℓth rock formation. The material parameters ρ_w and c_w are the mass density and specific heat, respectively, of the fluid. D_L is the fluid dispersion coefficient and λ_r is the thermal conductivity of the rock formation. Actually, the effect of the storage and dispersion on the heat extraction can be ignored based on the analysis in [7]. The fluid velocity in the fracture, v^ℓ, is expressed as

$$v^\ell = \frac{Q_i^\ell}{W \omega^\ell}, \quad at\, z = z_0^\ell (\ell = 1 \ldots N), \tag{2}$$

where Q_i^ℓ is the injection rate for the ℓth fracture. Considering the uniform distribution of injection rates, then we have

$$v^\ell = \frac{Q}{NW\omega}, \quad at\, z = z_0^\ell (\ell = 1 \ldots N). \tag{3}$$

where ω is the fracture aperture, assumed here to be spatially uniform and the same for each fracture.

2.2. Heat diffusion in the rock

The heat diffusion equation in each layer of the rock is $\kappa_r \nabla^2 T_r^\ell = \frac{\partial T_r^\ell}{\partial t}$, where the thermal diffusivity $\kappa_r = \lambda_r / \rho_r c_r$, ρ_r and c_r are the mass density and the specific heat capacity, respectively, of the rock formation. In the current model, the length and width of the fractures are large enough so that only the heat diffusion in the z direction must be considered, i.e.

$$\kappa_r \frac{\partial^2 T_r^\ell}{\partial z^2} = \frac{\partial T_r^\ell}{\partial t}. \tag{4}$$

2.3. Initial and boundary condition

The initial and boundary conditions for the whole system are listed as follows:

$$\begin{aligned}
&T_r = T_0 = A_0 z + B_0, && \text{on } t = 0, \\
&T_r^{\ell+1} = T_r^\ell = T_f^\ell, && \text{at } z = z_0^\ell, \\
&T_f^\ell = T_{\text{in}}^\ell, && \text{at } (0, z_0^\ell), \\
&T_r^{N+1} = B_0, && \text{at } z = 0.
\end{aligned} \tag{5}$$

3. Dimensionless formulation

By using the following the transformation

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad Z = \frac{z - H}{d}, \quad Z_0^\ell = \frac{z_0^\ell - H}{d}, \quad \tau = \frac{t\kappa_r}{d^2}, \quad \alpha = \frac{d}{H},$$

$$Pe = \frac{\rho_w c_w d (Q/NW)}{\lambda_r L},$$

$$\gamma = \frac{A_0 d}{A_0 H + B_0}, \quad \Theta_r^\ell = \frac{T_r^\ell - T^*}{T^*}, \quad \Theta_f^\ell = \frac{T_f^\ell - T^*}{T^*},$$

$$\Theta_{\text{in}}^\ell = \frac{T_{\text{in}}^\ell - T^*}{T^*}, \quad \Theta_0 = \frac{(A_0 z + B_0) - T^*}{T^*} = \gamma Z \tag{6}$$