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Reliability assessment of stability of underground rock caverns

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ABSTRACT

Conventional stability assessment of underground tunnels and caverns involves the determination of a factor of safety in which failure is assumed to occur when the load (stress) of the system exceeds the resistance. It is widely recognized that a deterministic analysis of the factor of safety gives only a partial representation of the true margin of safety, since the uncertainties in the design parameters affect the probability of failure. In this paper, a simplified procedure is proposed for evaluating the probability of stress-induced instability for deep underground rock caverns for preliminary design applications. Extensive parametric studies were carried out using a finite difference program to determine the factor of safety for caverns of various dimensions and rock mass strength. Subsequently, the limit state surface was determined through an artificial neural network approach following which a simplified reliability method of evaluating the probability of failure was developed.

1. Introduction

One of the major considerations in the design of an underground rock cavern is the evaluation of its stability since the excavation of the rock causes a redistribution of the stresses in the proximity of the underground opening. Various methods have been proposed to assess the cavern stability, and to assess the necessary support system to maintain the stability of the excavation. Common empirical methods include the use of rock classification systems such as the rock mass rating *RMR* [1] and *Q* methods [2]. Common numerical methods used to evaluate cavern stability can be categorized as continuum methods such as the Finite Element Method (FEM) [3] and Finite Difference Method (FDM) [4], and discontinuum methods such as the Distinct Element Method (DEM) [5] and the Discontinuous Deformation Analysis (DDA) [6]. The selection of a continuum or discontinuum approach depends on the size or scale of the discontinuities with respect to the size or scale of the problem that needs to be solved. There are no universal quantitative guidelines to determine when one method should be used instead of the other [7].

Conventional deterministic evaluation of stability of geotechnical structures and underground openings involves the use of a factor of safety *FS* which considers the relationship between the resistance *R* and the load (stress) *S*. The boundary separating the safe and failure domain is the limit state surface (boundary) defined as:

$$G(\mathbf{x}) = R - S = 0 \tag{1}$$

where $\mathbf{x}$ denotes the vector of the random variables. Mathematically, $R > S$ or $G(\mathbf{x}) > 0$ would denote a 'safe' domain, and $R < S$ or $G(\mathbf{x}) < 0$ would denote a 'failure' domain. For underground caverns, the limit state surface $G(\mathbf{x})$ is not known explicitly. Instead, it may be known only implicitly through a numerical procedure such as the finite element method. Therefore, the failure domain only can be found through repeated point-by-point numerical analyses with different input values. A closed-form limit state surface then is constructed artificially using polynomial regression methods.

However, polynomial regression models become computationally impractical for problems involving a large number of random variables and nonlinear limit state functions, particularly when mixed or statistically dependent random variables are involved. An alternative modeling technique is the use of neural networks. A neural network is a computer model whose architecture essentially mimics the knowledge acquisition and organizational skills of the human brain. A neural network consists of a number of interconnected neurons, which are logically arranged into two or more layers and interact with each other via weighted connections. These weights determine the nature and strength of the influence between the interconnected neurons. There is an input layer where data are presented to the neural network, and an output layer that holds the response of the network to the input. It is the intermediate layers, also known as hidden layers that enable these networks to represent and compute complicated associations between patterns. The back-propagation

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neural network (BPNN) learning algorithm is widely used because of the simplicity. The general objective of 'training' the neural network is to modify the connection weights to reduce the errors between the actual target outputs to a satisfactory level. This process is carried out through the minimization of the defined error function using the gradient descent approach. After convergence occurs (i.e. the errors are minimal), the associated trained weights of the model are tested with a separate set of testing data. This testing is used to assess the generalization capability of the trained model to produce the correct input–output mapping.

Even after obtaining the limit state surface, due to the uncertainties in the design parameters (random variables) in the limit state surface, it is impossible to predict the state of the system with accuracy. The alternative is to assess the probability of failure P_f. The calculation of P_f involves the determination of the joint probability distribution of R and S and the integration of the Probability Density Function (PDF) over the failure domain. For a problem with multiple n random variables, the calculation of P_f involves the determination of a multi-dimensional joint PDF of the random variables and the integration of the PDF over the failure domain.

A well-developed approximate alternative is to use the First-Order Reliability Method (FORM) [8]. Its popularity results from the mathematical simplicity, since only second moment information (mean and standard deviation) on the random variables is required to calculate the reliability index β. Some examples of the use of FORM in rock mechanics have been presented [9,10]. Mathematically, β can be computed [11] as

$$\beta = \min_{x \in F} \sqrt{\left(\frac{x_i - \mu_i}{\sigma_i}\right)^T [\mathbf{R}]^{-1} \left(\frac{x_i - \mu_i}{\sigma_i}\right)} \qquad (2)$$

in which x_i is the set of n random variables, μ_i is the set of mean values, $\mathbf{R}$ is the correlation matrix and F is the failure region. The minimization in Eq. (2) is performed over F corresponding to the region $G(\mathbf{x})=0$. Low and Tang [11] had shown that an EXCEL (Microsoft) spreadsheet environment can be used to perform the minimization and determine β. If the random variables have probability distributions close to normal, then P_f can be obtained from the expression:

$$P_f \approx \Phi(-\beta) \qquad (3)$$

in which $\Phi(-\beta)$ is the value of the cumulative probability. This value can be obtained from tables of the standard cumulative normal distribution function found in many textbooks or from built-in functions in most spreadsheets.

This paper utilizes the rock mass classification correlations and the numerical procedure known as the shear strength reduction technique to calculate the global factor of safety FS with regard to stress-induced instability. The BPNN is used to determine an empirical equation relating FS to the cavern dimensions B and H, as well as the rock mass quality Q. Charts based on this equation are presented for preliminary design purposes. A FORM spreadsheet is implemented with the neural network algorithm to calculate the reliability index (probability of failure) for cavern stability.

2. Numerical model of rock cavern

The FLAC^{3D} code (Itasca) was used to carry out the stability analyses of the underground rock caverns using the shear strength reduction technique. The shear strength reduction technique is available in many commercial finite element and finite difference programs. The technique has been applied to a number of geotechnical problems including rock caverns [12,13] and circular tunnels [14].

The procedure essentially involves repeated analyses by progressively reducing the shear strength properties until collapse occurs. For a Mohr–Coulomb material, by reducing the shear strength by a factor F the shear strength equation becomes

$$\frac{\tau}{F} = \frac{c}{F} + \sigma_n \frac{\tan\phi}{F} \qquad (4)$$

$$F = \frac{\tau}{c^* + \sigma_n \tan\phi^*} \qquad (5)$$

where τ is the shear strength, σ_n is the normal stress, and $c^* = \frac{c'}{F}$ and $\phi^* = \arctan\left(\frac{\tan\phi}{F}\right)$ are the new Mohr–Coulomb shear strength parameters. Systematic increments of F are performed until the finite element or finite difference model does not converge to a solution (i.e. failure occurs). The critical strength reduction value which corresponds to non-convergence is taken to be the factor of safety FS.

Only stress-induced failure was considered in this paper. In the FDM analyses, the three parameters that were varied were: the Tunneling Quality Index Q value, the cavern width B and cavern wall height H. Q cannot be directly used in the FLAC^{3D} calculations, though it is a commonly used quality index representing rock mass competence. In the analyses, the discontinuous nature of the rock is incorporated implicitly in the Mohr–Coulomb constitutive relationship used to represent the mass as an equivalent continuum. The rock mass properties are indirectly (through RMR) determined from the Q value by means of empirical equations as shown in Table 1. The Q value of each category and its corresponding Mohr–Coulomb rock properties to be used in the numerical calculation are shown in Table 2, in which the c, ϕ and E values are related to Q through the equations in Table 1. It should be noted that these relationships are intended to provide the initial estimates of the rock mass properties and should be used with caution in engineering design. It should also be pointed out that numerical analyses with the in-situ stress ratio K_0 in the range of 1–3 were also carried out. While the K_0 was found to significantly influence the state of stress and magnitude of displacements in proximity to the cavern, it had minimal influence on the FS and was therefore omitted as one of the design variables.

The cross-section of the cavern and boundary conditions are shown in Fig. 1. The cavern roof arc is semi-circular and the overburden height D from the ground surface to the top of the side wall is 100 m. The cavern length in the longitudinal direction is assumed as 1 m to simulate plane strain conditions. Outer boundaries are located far from the cavern to minimize the boundary effects. Full-face excavation is assumed in all analyses. Table 3 lists the design parameters and the values that were considered. Input file for each FLAC^{3D} execution includes a geometry model (B and H) and a mechanical model of Q-related rock mass properties.

For each numerical analysis, the safety factor FS was determined based on the strength reduction technique. Different combinations of Q, the cavern width B and the side cavern height

Table 1
Empirical equations relating Q with rock mass properties.

Properties	Equations	References
RMR from Q value	$RMR = 7\ln Q + 36$	[15]
Cohesion c (MPa)	$c(\text{MPa}) = 0.005(RMR-1)$	[1]
Friction angle ϕ (deg.)	$\phi = 0.5RMR + 4.5$	[1]
Deformation modulus E (GPa)	$E = E_m(\text{GPa}) = 2RMR - 100$ ($RMR > 50$) $E = E_m(\text{GPa}) = 10^{(RMR-10)/40}$ ($RMR \le 50$)	[16,17]