International Journal of Rock Mechanics & Mining Sciences 56 (2012) 88–99

Contents lists available at SciVerse ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Limiting envelopes of a dry porous limestone under true triaxial stress states

F. Descamps [a], M. Ramos da Silva [b], C. Schroeder [b], J-C Verbrugge [b], J-P Tshibangu [a,*]

[a] UMONS—University of Mons, Faculty of Engineering, Mining Engineering Department, 53 rue du Joncquois, 7000 Mons, Belgium
[b] ULB—Université Libre de Bruxelles, Soil Mechanics Laboratory, 87 avenue Buyl, 1050 Brussels, Belgium

ARTICLE INFO

Article history:
Received 29 February 2012
Received in revised form
22 June 2012
Accepted 22 July 2012
Available online 24 August 2012

Keywords:
Polyaxial
Rock
Limestone
Yield surface
Intermediate principal stress
Octahedral sections
Meridian sections

ABSTRACT

The present study investigates the influences of the confining pressure and the intermediate principal stress on the mechanical behaviour of a porous limestone. True triaxial tests are carried out on a broad range of confining stresses covering both brittle and ductile regimes. The experiments included the special cases of triaxial compression, extension and isotropic compression, as well as tests at constant levels of the intermediate principal stress. The resulting stress–strain curves and yield stresses are then discussed. Three-dimensional yield envelopes are subsequently built in meridian and octahedral planes. The shape of the octahedral sections changes with the mean stress from triangular to hexagonal and then quasi circular. At even higher mean stresses, the evolution continues towards a triangle with apexes oriented in the direction of triaxial extension stress states. Various existing yield criteria are fitted and compared to the experimental data. Finally, a three-dimensional yield surface is proposed combining the previous observations.

1. Introduction

Many geotechnical or geological processes require a thorough understanding of the mechanical properties of porous rocks. In petroleum-related applications, for instance, these rocks form part of oil reservoirs and raise problems such as sand production or compaction [1–3]. As a result, their complex behaviour has been the subject of extensive research, involving numerous triaxial compression tests ($\sigma_1 \geq \sigma_2 = \sigma_3$, stresses being reckoned positive in compression) carried out on several types of sandstones [4–10] or carbonate rocks [11–18]. A typical feature of the behaviour of such materials, as evidenced by these studies, is the marked effect of the confining pressure ($P \equiv \sigma_2 = \sigma_3$) on their mechanical properties. At low confining pressures, the increase of the major principal stress leads to brittle behaviour. Localized shear failure is associated with strain softening, as well as dilatancy as the samples approach peak stress. Rock strength increases with confining pressure, as does the onset of dilatancy.

At higher confining pressures, the samples enter the ductile regime, with significant strain hardening and compaction as σ_1 is raised; the onset of yield is a decreasing function of the confining pressure. If σ_1 is further increased, some experimental results reveal that the material can progressively evolve to a dilatant yielding mechanism [16] or even to brittle failure [19]. The main mechanisms governing ductile deformation include pore collapse, grain crushing or crystal plasticity (the latter being more likely to occur in carbonate rocks [20]) and are accompanied by a decrease in permeability and possibly an increase in acoustic emission activity (especially in sandstones [21]). The rock's behaviour gradually evolves from brittle to ductile in a transitional range of confining pressures, whose extent may depend on the rock type. In this transitional zone, rock samples may exhibit strain softening or perfectly plastic deformation with virtually no dilatancy.

The previous observations give detailed information about the effect of confinement, but are limited, in principle, to the stress states where $\sigma_2 = \sigma_3$. In many situations, however, the in situ stress field is reported to be anisotropic ($\sigma_1 > \sigma_2 > \sigma_3$), for instance in boreholes [22], in tunnels or other excavations [23,24], and more generally in rock masses at shallow to intermediate depths [25]. This requires an evaluation of the separate effect of the intermediate principal stress on the mechanical properties.

One of the main goals of this paper will be to evaluate this influence in the case of a porous limestone. This will be done by means of true triaxial (or polyaxial) tests conducted in order to cover a wide a range of principal stresses ranging from the brittle to ductile regimes.

2. Influence of the intermediate principal stress

2.1. Polyaxial testing

Several procedures may be considered to investigate the influence of the intermediate principal stress. A common method is to compare

* Corresponding author. Tel.: +32 65 37 45 18; fax: +32 65 37 45 20.
E-mail address: katshidikaya.tshibangu@umons.ac.be (J.-P. Tshibangu).

1365-1609/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijrmms.2012.07.013

triaxial compression and carefully calibrated triaxial extension tests ($\sigma_1 = \sigma_2 \geq \sigma_3$), as reported by Wu and Kolymbas [26], Hickman [27], and Dehler and Labuz [28]. Since compression and extension represent the two limiting values of σ_2 for a given value of σ_3, the differences between the two experiments may be attributed to the influence of the intermediate principal stress. Further insight can also be gained from experiments in which $\sigma_1 \geq \sigma_2 \geq \sigma_3 = 0$ [29,30], which can in turn be compared with the two previous stress states [31]. However, in order to study the full range of variations of σ_2, specially designed apparatus is to be used, capable of applying three independent principal stresses to the rock specimens [32]. Some devices involve three-dimensional loading of rectangular prismatic or cubical specimens [25,33–36] with different types of boundary conditions [37].

Comparative studies led to the conclusion that stress strain curves, failure mode, or strength could be markedly affected by these boundary conditions [38]; devices minimizing end friction or flexible boundary conditions are to be recommended to give reproducible results. Other devices apply stresses to hollow cylinder specimens with a combination of external and internal pressures, axial load and/or torque [39–41]. Other specific designs include the cell designed by Smart et al. [42], or the confined Brazilian disc presented by Jaeger and Hoskins [43].

2.2. Available experimental results

Using the experimental data obtained from these polyaxial tests, the influence of the intermediate principal stress on strength can be investigated by plotting, for a given value of σ_3, the major principal stress at failure versus σ_2 [23,25,32,35,44,45]. The experimental evolutions were seen to vary from one rock type to another. Several results first show a marked increase of strength with σ_2 at lower values of the intermediate principal stress. This increase stabilizes until a plateau is eventually reached. The strength may then slightly decrease with σ_2 or remain constant to reach the value of triaxial extension, the latter still being higher than the strength measured in compression [23,25,46]: this evolution will be referred to as type I below. On other rock types (type II), however, the intermediate principal stress may have little or no influence on strength [23,47]. At higher values of the minor principal stress, σ_2 may affect the brittle–ductile transition. It was indeed shown by Heard, as cited by Paterson and Wong [48], and Mogi [31] that the brittle–ductile transition in Solnhofen limestone occurred at higher confining stresses in triaxial extension than in triaxial compression. Mogi [32] and Michelis [49] later confirmed under more general stress states that an increase in σ_2 led to an embrittlement of the rock.

The intermediate principal stress also influences the measured stress–strain curves. Michelis [49] and Takahashi and Koide [23] showed that expansion increased towards the σ_3 direction, and decreased in the σ_2 direction as the intermediate principal stress increased; these observations were made for σ_2 reaching up to about 1/3 and even in some cases 1/2 of σ_1. At higher values of σ_2, it is expected that ε_2 will rise in the opposite direction leading to $\varepsilon_2 = \varepsilon_1$ in triaxial extension. These changes in the measured strains reduce specimen dilatancy as the intermediate principal stress increases [23,25]; the onset of dilatancy occurs at higher major principal stresses. It is to be recalled that this decrease in dilatancy corresponds to more brittle behaviour, which contrasts with the conventional triaxial testing interpretations mentioned earlier, where decreasing dilatancy is associated with more ductile behaviour.

2.3. Three-dimensional representation

The influence of the intermediate principal stress may alternatively be represented using the three stress invariants (q, θ, p) defined below [50–52]:

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = \sqrt{\frac{2}{3}J_2} = \frac{\sqrt{2}}{3}q \tag{1}$$

$$\theta = a\tan\left(\sqrt{3}\frac{\sigma_2-\sigma_3}{(\sigma_1-\sigma_2)+(\sigma_1-\sigma_3)}\right) = \frac{1}{3}a\cos\left(\frac{3\sqrt{3}J_3}{2J_2^{3/2}}\right) \tag{2}$$

$$z = \sqrt{3}p = \frac{\sqrt{3}}{3}(\sigma_1+\sigma_2+\sigma_3) = \frac{\sqrt{3}}{3}I_1 \tag{3}$$

where τ_{oct} is the octahedral shear stress, q is the deviatoric stress, θ is the Lode angle, p is the mean stress; I_n and J_n denote the nth invariant of the stress and deviatoric stress tensors, respectively. In this work, the convention for the Lode angle is therefore 0° in triaxial compression and 60° in triaxial extension.

Alternatively, the inverse relationships may be derived as:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{Bmatrix} = \begin{Bmatrix}p\\ p\\ p\end{Bmatrix} + \frac{2}{3}q\begin{Bmatrix}\cos\theta\\ \cos(2\pi/3-\theta)\\ \cos(2\pi/3+\theta)\end{Bmatrix} \tag{4}$$

These invariants define a cylindrical coordinate system (τ_{oct}, θ, $z=\sqrt{3}p$) in the (σ_1, σ_2, σ_3) space, also known as Haigh–Westergaard coordinates. Two families of planes may be defined in this coordinate system: octahedral or deviatoric sections (p=constant) and meridian sections (θ=constant). These two planes may be used to represent three-dimensional states of stress.

Observations on several rock materials indicate that the shape of the brittle failure surface in octahedral sections generally evolves from a triangle to a circle as the mean pressure increases [53,54]. This was noted in particular on Soignies limestone (a compact rock [55]) and Adamswiller sandstone (of about 20% porosity [56]). A similar trend is observed for concrete [57,58] and soils [59]. Bigoni and Piccolroaz [60] also report analogous evolutions for metal powders submitted to compaction as they evolve from granular materials to cohesive and porous metals.

Data given by Mogi [31] suggests that the brittle to ductile transition occurs at higher mean pressures in extension than in compression. In the ductile regime, Zhu et al. [61] showed for three sandstones that the yield points corresponding to triaxial compression and extension fell on a single surface in the meridian plane.

3. Yield criteria in three dimensions

The previously mentioned experimental observations have been modelled using the theory of elastoplasticity. Many yield or failure criteria have indeed been developed in the past decades describing a wide range of shapes and have been extensively reviewed by Yu [62] for various types of materials and for instance by Mestat [63] for geomaterials.

Several of these criteria are based on phenomenological aspects: sliding friction for the Mohr–Coulomb criterion or stored energy threshold for the Drucker–Prager criterion [64]. Others are based on micromechanical considerations: crack propagation in the context of fracture mechanics for the Griffith criterion, or energy dissipated during sliding of oriented cracks in the Wiebols and Cook approach [65]. Finally, the development of laboratory devices and the accumulation of available experimental data have led to the development of various empirical yield functions. Such criteria include, among others, the functions proposed by Hoek and Brown [66], Kim and Lade [54] or Willam and Warnke [57].

The previously mentioned criteria that will be used in the next sections are briefly summarized in Table 1, which gives the