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Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of jointed rocks

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ABSTRACT

Rocks encountered in civil and mining engineering fields are invariably jointed and act under triaxial or polyaxial stress conditions. The Mohr–Coulomb shear strength criterion is the most widely used criterion for jointed rocks. In its present form there are two major limitations of this criterion; firstly it considers the strength response to be linear, and, secondly the effect of the intermediate principal stress on the strength behaviour is ignored. A modified non-linear form of Mohr–Coulomb strength criterion has been suggested in this study to overcome these limitations. Barton's concept of critical state for rocks has been imbibed in the linear Mohr–Coulomb criterion to deduce a semi-empirical expression for non-linear criterion. However, the shear strength parameters of the conventional Mohr–Coulomb criterion are used in the proposed criterion. The proposed criterion is a simple and rational nonlinear polyaxial strength criterion for anisotropic jointed rocks. In an earlier publication [1] the applicability of the criterion was evaluated for intact rocks. In present paper the criterion is extended to jointed rocks, which are anisotropic in nature. The applicability of the proposed criterion has been verified by applying it to extensive experimental data on triaxial and polyaxial test results on jointed rocks available from literature. Applicability of the criterion, to explain rock burst conditions for some Indian rocks, is also demonstrated.

1. Introduction

Rocks encountered in civil and mining engineering applications are, in general, jointed and anisotropic in nature. Their strength, under prevailing confining stress conditions, is needed while analysing problems related to deep tunnels, underground excavations and foundations. The strength of jointed rock, as a whole, depends on strength of the intact rock, joint geometry and surface characteristics of the joints. Depending on joint geometry and joint strength characteristics the rock blocks may undergo sliding, shearing, splitting or rotation at the time of failure. Substantial research has been carried out in past to understand the mechanical behaviour of rock joints [2–6]. The outcome of these studies may be used to analyse the rock mass behaviour if the joints are modelled explicitly.

Classification approaches [7–8] consider the rock mass as an equivalent continuum, and the effect of the joints is considered implicitly. These approaches have found wide acceptability in the field. Laboratory studies on rocks and model materials have also been used to represent rock mass as an isotropic or anisotropic equivalent continuum [9–13]. These equivalent continuum approaches can be used to characterize the rock mass, from which the rock mass strength under unconfined state may be obtained. The effect of confinement (triaxial or polyaxial) may then be included using an appropriate strength criterion. The main objective of the present study is to suggest an approach in which the effect of minor and intermediate principal stress on the strength of jointed rock mass can be obtained with adequate accuracy at any given confining pressure.

The strength behaviour of the rocks is generally expressed by a strength criterion. Mohr–Coulomb strength criterion is the most widely used criterion for intact and jointed rocks as well. As discussed in earlier publication [1], the criterion in its present form suffers from two major limitations: (i) it ignores the non-linearity in strength behaviour, and (ii) the effect of intermediate principal stress is not considered in its conventional form. A non-linear strength criterion for intact rocks was suggested by Singh et al. [1], which is an extended form of the conventional Mohr–Coulomb criterion. The intermediate principal stress was also incorporated in the criterion. Using available extensive data from triaxial and polyaxial tests it was shown that the proposed simple criterion works better than the other popular criteria in vogue. An important advantage of the proposed criterion is that the conventional Mohr–Coulomb shear strength parameters are retained as such. In present paper, the criterion proposed for intact rock [1] is extended to jointed rocks. The applicability of

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the proposed criterion has been verified by applying it to data base available from literature.

2. Triaxial conditions

2.1. Modified Mohr–Coulomb criterion

The complete derivation of the criterion for intact rocks has already been presented in [1]. The criterion was deduced from Barton's concept of critical state in rocks [14]. Barton [14] states that *"critical state for any intact rock is defined as stress condition under which Mohr-envelope of peak shear strength of the rocks reaches a point of zero gradient. This condition represents the maximum possible shear strength of the rock. For each rock, there will be a critical effective confining pressure above which the shear strength cannot be made to increase"*. Modified Mohr–Coulomb criterion for intact rock [1], in its general form, was expressed as

$$(\sigma_1-\sigma_3)=\sigma_{ci}+\frac{2\sin\phi_{i0}}{1-\sin\phi_{i0}}\sigma_3-A'\sigma_3^2 \quad \text{for } 0\le\sigma_3\le\sigma_{crti} \tag{1}$$

where σ_3 and σ_1 are the effective minor and major principal stresses at failure; σ_{ci} is the UCS of the intact rock, defined as

$$\sigma_{ci}=\frac{2c_{i0}\cos\phi_{i0}}{1-\sin\phi_{i0}} \tag{2}$$

where c_{i0} and ϕ_{i0} are the Mohr–Coulomb shear strength parameters obtained by conducting triaxial strength tests on rock specimens at low confining pressures ($\sigma_3\to 0$); A' is an empirical constant for the rock type under consideration, and σ_{crti} is the critical confining pressure for the rock. By employing critical state concept [14], Eq. (1) was reduced to the following form:

$$(\sigma_1-\sigma_3)=\sigma_{ci}+\frac{2\sin\phi_{i0}}{1-\sin\phi_{i0}}\sigma_3-\frac{1}{\sigma_{crti}}\frac{\sin\phi_{i0}}{(1-\sin\phi_{i0})}\sigma_3^2 \quad \text{for } 0\le\sigma_3\le\ \sigma_{crti} \tag{3}$$

or

$$(\sigma_1-\sigma_3)\ =\sigma_{ci}+\frac{2\mu_{i0}}{\sqrt{1+\mu_{i0}^2}-\mu_{i0}}\sigma_3-\frac{1}{\sigma_{crti}}\frac{\mu_{i0}}{\sqrt{1+\mu_{i0}^2}-\mu_{i0}}\sigma_3^2 \quad \text{for } 0\le\sigma_3\le\sigma_{crti} \tag{4}$$

where μ_{i0} is the coefficient of internal friction ($\tan\phi_{i0}$) of the rock under low stresses.

The critical confining pressure will depend on the particular rock type and its lithology. However statistical analysis of more than 1100 triaxial test data in [1] has shown that the critical confining pressure for an intact rock can be taken nearly equal to its UCS without introducing error of engineering significance in the prediction of confined strength. The modified Mohr–Coulomb criterion for intact isotropic rocks under triaxial stress condition may therefore be expressed as

$$(\sigma_1-\sigma_3)=\sigma_{ci}+\frac{2\sin\phi_{i0}}{1-\sin\phi_{i0}}\sigma_3-\frac{1}{\sigma_{ci}}\frac{\sin\phi_{i0}}{(1-\sin\phi_{i0})}\sigma_3^2 \quad \text{for } 0\le\ \sigma_3\ \le\ \sigma_{ci} \tag{5}$$

Brown [15] conducted a number of triaxial tests on intact and jointed rock mass specimens under triaxial stress condition. Fig. 1 (redrawn from Brown [15]) shows a plot indicating variation of shear stress at failure with normal stress for intact and jointed rock. It is observed from this plot, that the failure envelopes for jointed and intact rocks tend to merge with each other at sufficiently high confining pressure (zone 3, Fig. 1). The failure envelope for the intact rock represents the upper limit of all the failure envelopes. Non-linearity in strength behaviour at low σ_3, is very high in case of jointed rock as compared to the intact rock. Consequently, the difference between the strength values of intact and jointed rock is high at low confining pressures. As confining pressure increases, the difference between the strength of the intact and the jointed rock decreases. At sufficiently high confining pressure this difference tends to reduce to almost zero. In other words beyond this confining pressure, the effect of joints ceases to exist, and the jointed rock behaves like an intact rock; as the intact rock follows critical state concept, the jointed rock will also follow the same.

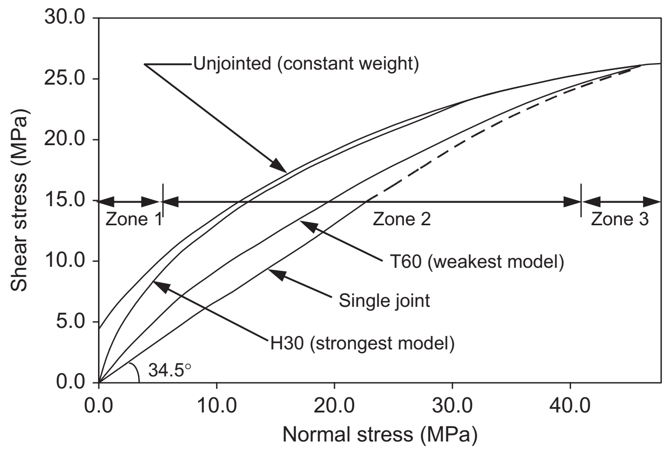

Fig. 1. Mohr envelopes for intact and jointed specimens (redrawn from [16]).

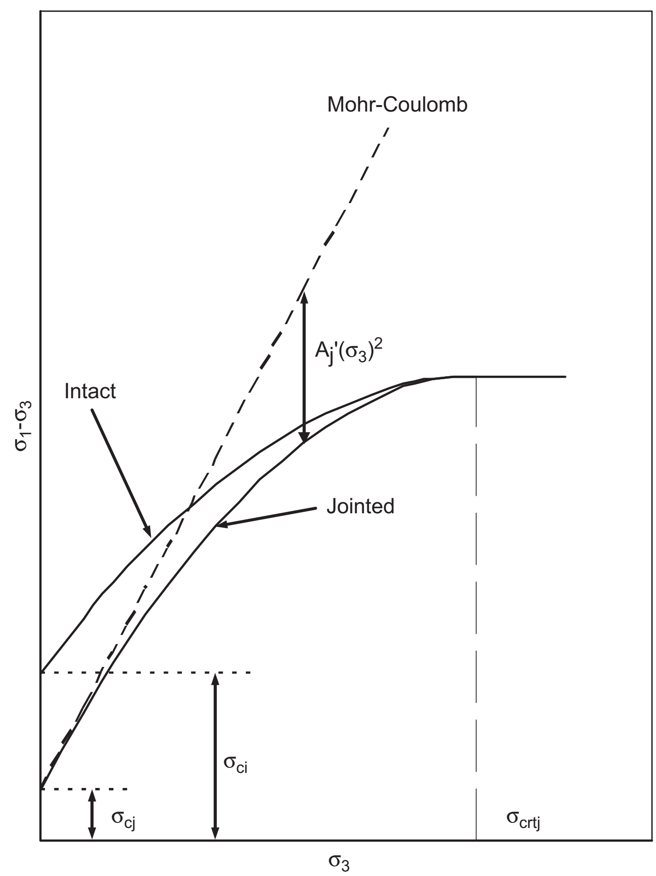

Fig. 2. Non-linear variation of strength of intact and jointed rock.

Taking the idea from the above discussion, a model for strength behaviour of jointed rocks is shown in Fig. 2. The strength of jointed rock is assumed to increase with confining pressure following a parabolic variation and the strength envelope merges with that of intact rock at a sufficiently high confining