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Reliability-based design for allowable bearing capacity of footings on rock masses by considering angle of distortion

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ABSTRACT

This study addresses the impact of spatial variability on the angle of distortion between two footings in rock masses. A simple elasto-perfectly-plastic model based on the Hoek–Brown criterion is taken to simulate the spatial variation of rock mass properties in the finite element analyses. This model is calibrated by a large rock mass database. With Monte Carlo simulations, stochastic samples of angle of distortion between two footings are obtained, which are further used to derive reliability-based allowable bearing stresses. The analysis results show that the geological strength index (GSI) of rock masses and uniaxial compressive strength of intact rock are the two dominate factors that affect the reliability-based design. Comparisons to the existing codes show that these codes are appropriate for poor to fair rock masses, conservative for good to very good rock masses and un-conservative for very poor rock masses.

1. Introduction

Intact rocks are usually considered as competent materials in terms of the strengths and deformability. However, fractured rock masses can behave quite differently from intact rocks. In particular, unacceptable settlement may occur for rock masses that are highly fractured and highly varying in space. Behaviors of foundations on spatially variable soils have been studied for years. For stability analyses, Cherubini [1] considered the ultimate bearing capacity on an idealized c–ϕ soil, incorporating spatial averaging effect. Popescu et al. [2] implemented full random field finite element analyses, considering spatial correlation. More recently, Cho and Park [3] further investigated the effect of cross-correlation between the random fields of various soil properties. For deformation analyses, Fenton and Griffiths [4], Nour et al. [5] and Houy et al. [6]studied differential settlements, while Fenton and Griffths [7] considered three-dimensional analyses, modeled by three-dimensional random fields of elastic modulus. Breysse et al. [8] considered the impact of soil–structure interaction in random fields.

Quantification of spatial variabilities in shear strengths and deformability properties of rock masses is challenging. This may be due to the following reasons: (a) statistical data for spatially variable rock masses, including the coefficients of variation and scales of fluctuation, are scarce; (b) rock masses are mechanically complicated, partially due to the presence of joints, beddings or even faults. For shear strengths of rock masses, Hoek and Brown [9] proposed the well-known Hoek–Brown failure criterion for heavily fractured rock masses. It is parameterized by three parameters, including the geological strength index (GSI), and the Hoek–Brown's constant m_i, and the uniaxial compression strength σ_{ci} for intact rock. Efforts to quantifying these three parameters have been taken: Hoek et al. [10,11] provided charts to quantify the range of GSI based on the appearance (structures and surface conditions) of the rock masses. Marinos and Hoek [12] summarized a table of possible ranges for the m_i parameter for various rock types. This table is reproduced herein in Table 1.

For deformability of rock masses, the deformation modulus of rock masses E_m is correlated to rock mass rating (RMR) [14–18], to rock quality index (RQD) [19,20], to Q index [21], and to GSI [13,22]. Among them, Hoek and Diederichs [13] compiled a large database for E_m based on field tests conducted in China and Taiwan. They further proposed a GSI-based correlation model for predicting E_m.

Based on the aforementioned results and database, this study aims to construct a random field model for the spatially variable shear strengths and E_m of highly fractured rock masses, and subsequently to conduct probabilistic analyses for angle of distortion between two footings on rock masses. Efforts are made to quantify the variability in shear strengths and E_m, including

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Table 1
Typical m_i and modulus ratio (MR) values, and their ranges for various rock types (ranges adopted from [12,13]).

Sedimentary/(m_i)/[MR]	Metamorphic/(m_i)/[MR]	Igneous/(m_i)/[MR]
Sandstones (17 ± 4) [200–350]	Marble (9 ± 3) [700–1000]	Granite (32 ± 3) [300–550]
Siltstones (7 ± 2) [350–400]	Gneiss (28 ± 5) [300–750]	Tuff (13 ± 5) [200–400]
Claystone (4 ± 2) [200–300]	Schists (12 ± 3) [250–1100]	
Shales (6 ± 2) [150–250]		
Limestone (11 ± 4) [400–1000]		
Chalk (9 ± 3) [100+]		

their scales of fluctuation. Random field finite element analyses are further conducted to simulate the angle of distortion of the footings. Based on the simulation results, reliability-based design tables and charts for allowable bearing stress are developed.

2. Random field models

Spatial variabilities of rock properties can be modeled as random fields. Among random field models, stationary lognormal random fields are widely used for rock properties due to their simplicity and also due to the fact that most rock properties are non-negative. A stationary lognormal random field can be simulated by taking exponential of a stationary Gaussian random field, which can be in turn simulated by adding a desirable shift term to a zero-mean stationary Gaussian random field $\xi(h,z)$, where h and z denotes horizontal and vertical coordinates, respectively. The two-dimensional (2-D) zero-mean stationary Gaussian random field $\xi(h,z)$ can be characterized by its standard deviation σ_ξ and auto-correlation function $\rho_\xi(\Delta h,\Delta z)$.

2.1. Auto-correlation function

The auto-correlation function of a 2-D stationary random field $\xi(h,z)$ is defined to be the correlation between two locations with horizontal distance of Δh and vertical distance of Δz:

$$\rho_\xi(\Delta h,\Delta z) \equiv \frac{COV(\xi(h,z),\xi(h+\Delta h,z+\Delta z))}{\sqrt{VAR(\xi(h,z))}\sqrt{VAR(\xi(h+\Delta h,z+\Delta z))}} \quad (1)$$

where VAR denotes the variance, and COV denotes covariance. In the case that the vertical and horizontal variabilities are decoupled, a popular 2-D auto-correlation model is as follows [23]:

$$\rho_\xi(\Delta h,\Delta z) = \exp\left(-\frac{2|\Delta h|}{\delta_h} - \frac{2|\Delta z|}{\delta_z}\right) \quad (2)$$

where δ_h and δ_z are the horizontal and vertical scales of fluctuation (SOF), respectively. The SOF is sometimes called the correlation length: rock properties of two locations with distance less than the correlation length are correlated, and vice versa. Moreover, the magnitude for correlation decreases as the distance increases. This manifests the common observations in natural rocks: properties are strongly correlated within a small interval and are weakly correlated for a large interval.

2.2. Simulation of zero-mean stationary Gaussian random fields

Simulations of zero-mean stationary Gaussian random fields $\xi(h,z)$ over a rectangular domain of size $L_h \times L_z$ can be achieved through the Fourier series expansion as follows:

$$\xi(h,z) = \mathrm{Re}\left(\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(a_{mn}+ib_{mn})\exp\left(\frac{i2m\pi h}{L_h}+\frac{i2n\pi z}{L_z}\right)\right) \quad (3)$$

where *Re* denotes the real part of the enclosed complex number, and a_{mn} and b_{mn} are independent zero-mean Gaussian random

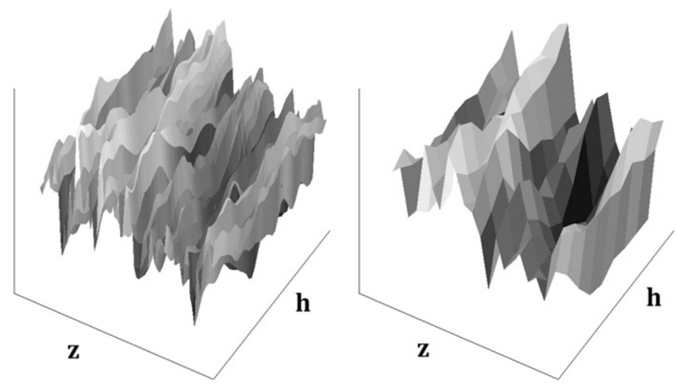

Fig. 1. Realization of $\xi(h,z)$ (left plot) and its local averaging over the elements (right plot: the domains of the elements are shown in the z–h plane).

variables with the following variance:

$$\sigma_{mn}^2 = \frac{\sigma_\xi^2}{L_h L_z}\left[\frac{\delta_h - \exp(-L_h/\delta_h)\,\delta_h(-1)^m}{1+m^2\pi^2\delta_h^2/L_h^2}\right]\left[\frac{\delta_z - \exp(-L_z/\delta_z)\,\delta_z(-1)^n}{1+n^2\pi^2\delta_z^2/L_z^2}\right] \quad (4)$$

(See Appendix A for the derivations of this variance.) This variance term depends on the standard deviation σ_ξ and the SOFs. The real part of Eq. (3) is a sample of the random field $\xi(h,z)$. It is not necessary to sum up the infinite terms in Eq. (3) because σ_{mn}^2 usually decays very rapidly with increasing $|m|$ and $|n|$. In this study, we found that it is only necessary to sum up to an $|m|$ or $|n|$ value corresponding to

$$\frac{1}{L}\left[\frac{\delta - \exp(-L/\delta)\,\delta(-1)^{|m|}}{1+m^2\pi^2\delta^2/L^2}\right] \approx 10^{-5} \quad (5)$$

without noticeable errors.

2.3. Simulation of local averaging in finite elements

The above equations are useful for generating samples of continuous function $\xi(h,z)$. However, in finite element analysis, a constant average value should be assigned to each element. A procedure called the local average subdivision (LAS) of achieving the "local averaging" over each element is developed by Fenton and Vanmarcke [24]. This procedure requires fairly complicated steps and is limited to finite element meshes with equally spacing rectangular elements. Fenton and Griffiths [25] further suggested to conduct the local averaging in the logarithm space of geotechnical parameters rather than in the original space, i.e. conducting the local averaging in the $\xi(h,z)$ space. It is then necessary to average the sampled $\xi(h,z)$ over an element area to obtain the average property for that element. The effect of local averaging is illustrated in Fig. 1: the left plot is a realization of the continuous function $\xi(h,z)$, while the right plot is its local averaging over the elements.

Suppose the averaged property $\xi_{\Delta h\times\Delta z}(h_e,z_e)$ is desirable for a rectangular element defined by the $[h_e-\Delta h/2, h_e+\Delta h/2]$ and