International Journal of Rock Mechanics & Mining Sciences 47 (2010) 553–558

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Optimized mesh generation for two-dimensional finite element analysis of underground excavations in rocks masses traversed by joints

A.M. Zsaki

Department of Building, Civil and Environmental Engineering, Concordia University Montreal, Quebec, Canada

ARTICLE INFO

Article history:
Received 8 April 2009
Received in revised form
16 December 2009
Accepted 4 March 2010

Keywords:
Mesh optimization
Excavation design
Jointed rock mass
Stress analysis

ABSTRACT

Deformation and distribution of stresses around underground excavations in jointed rock masses are influenced by yield and slip along joints. Since stresses are often computed using non-linear finite element analysis, and considering the excavation sequence, a number of what-if scenarios are performed, requiring considerable time. Often an accurate solution is only required in a limited part of a larger model. While mesh optimization can reduce the number of elements and analysis effort, the previously published optimization method by the author only considered the effect of excavations at a region of interest. The method was based on a cost function to prioritize the automatic removal of geometric detail from the boundary representation of the domain and the ensuing mesh. The inclusion of the effects of joints and excavation interaction in a stress analysis model, as part of the mesh optimization process, is developed in this paper with an application to a representative stress analysis case. The resulting optimized meshes can represent the displacement and stresses at the region of interest with considerable accuracy while capturing the yield along joints in the rock mass.

1. Introduction

The design of underground excavations constructed in rock masses traversed by a few major joints is often dictated by the yield and the subsequent stress redistribution in the vicinity of a joint. The amount of failure and yield, both along a joint and around the excavations, dictates the type of support system used. Often some form of numerical stress analysis is performed to gain insight into the displacement, yield and stress distribution. With the incorporation of material non-linearity in the constitutive models for both the rock mass and the joint, these analyses could take considerable time and be taxing on computational resources. The Finite Element Method (FEM) [1], by the discretization of the governing differential equations, is a prevalent tool used in modeling of the rock mass and joint behaviour. The FEM requires that a computational domain to be subdivided (discretized) into elements. Potentially, there could be an unlimited number of combinations for the arrangement of finite elements, forming a mesh, which could be used to arrive at a solution of the problem. Subsequently, the type of element formulation can be chosen as well. Generally, results obtained from an analysis could show considerable dependence on the mesh density and the choice of element formulation [2]. It is well accepted that by increasing the number of elements or by using higher order elements (h- and p-type refinements) the solution accuracy increases [1]. However, this type of refinement is often intrinsic to the problem being analyzed, e.g. the refinement occurs where there is an 'interesting' development in the solution. The refinement process requires re-computation of the problem until a convergence criterion is satisfied. In quite a few circumstances, this approach leads to a better understanding of the behaviour of the model on the global scale. Often much of the computational cost is associated with this phase of the solution. Depending on the size of a model being analyzed and the number of excavations, the joint yield can disturb the stress field only in the vicinity of the failed region not affecting the model globally. Similarly, depending on the relative size and separation of excavations and joints, the interaction between excavations and joints can range from considerable overstress in the rock mass in between them to no influence at all. Thus, it would be beneficial to selectively simplify or optimize a model to obtain a finite element mesh that captures the local response of rock mass yet has the least amount of elements to speed up solution. Particularly, if the analyst is interested in the displacements or stresses or the rock support behaviour only in a part of a larger model (development of a new drift, shaft or a tunnel, etc.). This paper further gives the ideas developed previously [3] to incorporate the treatment of joints in a model as part of an automatic mesh optimization strategy to reduce the number of finite elements required while maintaining the quality of a solution. In the context of this paper, mesh optimization is defined as the automatic generation of an optimized finite

E-mail address: zsaki@encs.concordia.ca

1365-1609/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijrmms.2010.03.005

element mesh with respect to the choice of the location of a 'region of interest' (ROI) within the model. In essence, the optimization process selectively coarsens a previously discredited boundary representing excavations, geologic boundaries and joints using a cost function but without any user intervention. The 1D discretization serving as the input can be generated using any traditional mesh generator. The optimization process presented is different from other 'mesh optimization' (coarsening/refining) strategies that work on a 2D mesh in that it operates on a higher level of abstraction. While other mesh optimization methods thrive to improve the quality of a mesh, the method developed in this paper tries to capture the intent of the analyst by optimizing the density and location of vertices (1D discretization) based on what is important or interesting, hence the ROI concept. Any 2D mesh generated from these optimized 1D discretizations will result in mesh densities optimal in ensuring a quality solution at the ROI and new for this paper, in the vicinity of the joints.

2. Related research

Mesh optimization strategies have been used in FEM analysis to reduce geometric detail contained in a CAD model prior to meshing to actually make it possible to generate a quality FEM mesh [4,5], with an inherent benefit of reducing the number of finite elements. This type of mesh optimization results in a global reduction of small geometric detail, which often contributes a negligible amount to the distribution of stress within a part being analyzed. Other methods for generating and optimizing meshes, often involving reducing, or coarsening a mesh for use in FEM or boundary element analysis, were investigated and a few notable algorithms were developed. One such algorithm [6], for application in fracture mechanics, attempted to improve element quality during the meshing process, and thus create an optimum mesh. Similarly, for many problems successive iterations of solution could be required, particularly with a consecutive refinement or coarsening the mesh, such as found in [7] for 2D and in [8] for 3D applications. These algorithms, often referred to as adaptive methods, can easily modify the local distribution of mesh density to reflect an 'interesting' development of the solution and accelerate convergence. Other algorithms, such as presented in [9] attempt to generate an optimum mesh as measured by common mesh quality metrics such as maximum internal angle or edge length ratios. Common to the above methods is that they either endeavor to generate an optimum mesh, based on geometric metrics or successive iterations of a solution. However, neither of them takes into consideration an *a priori* knowledge of how the solution should be, where large stresses are expected or where the analyst requires an accurate solution. Similarly, the problem of mesh density and complexity contained in models used in online games poses a considerable issue since the transmission over a network is a direct function of mesh size [10]. Thus any appearance-preserving mesh optimization should reduce transmission time. Alike to the CAD model simplification, only the overall geometry of the model used in games is optimized.

Exploiting the concept of 'excavation disturbed zone' (EDZ) and the ROI concept, an algorithm was developed to aid the generation of a finite element mesh to reduce the number elements and computation time while providing an optimum solution at the ROI [3]. However, that previous algorithm did not incorporate joints in the model and was unable to exploit the potential for simplification for models in jointed rock masses. As a natural evolution of the algorithm, inclusion and optimization of the discretization of joints is presented in this paper.

3. Brief summary of mesh optimization using the ROI concept

The salient features of the mesh optimization method based on proximity to excavations and inter-excavation interaction is presented, however, the detailed treatment is already published in [3]. In this paper the terms 'mesh' and 'discretization' can be used interchangeably, but 'discretization' is more often refers to the subdivision of a polygonal geometry into 1D line segments whereas mesh is more of a 2D assemblage of elements. Similarly, the 'geometric detail' refers to the discretization of a feature, such as an excavation, such that it represents its shape. The optimization method operates on the discretized geometry; its results are equally applicable to both 2D finite element and boundary element analyses as well.

At the heart of the optimization method is a correlation of geometric detail contained in a model to its effect on the stress distribution around excavations and joints. It is appreciated that the shape of an excavation directly affects the stress field around it. Yet, if the stress field is investigated at some distance from an excavation, due to the extent of EDZ, less and less fine geometric detail is required to arrive at a 'satisfactory' stress solution. By 'satisfactory' solution, the results are interpreted in the context of the data-limited nature of modeling in geomechanics, where if the solution is within 20% of the 'true' value it is deemed acceptable [11]. Perhaps, as discussed in [3], the hardest part of the mesh optimization is to qualitatively unite geometric level of detail with the resulting state of stress. The input to the optimization algorithm is a finely discretized boundary geometry in 1D representing excavations, joints and geologic boundaries. Thus, both polygonal and parametric representations can serve as the input provided they can be discretized along their length via most existing mesh generators.

Nevertheless, the first step in the algorithm is to map out the extent of disturbance caused by an excavation. Using an ellipse to approximate the geometry of an excavation of arbitrary 2D geometry, a closed form solution, such as presented by Bray [12] is used to establish an EDZ, as measured by the percent deviation from the existing field stress. By keeping the maximum value of

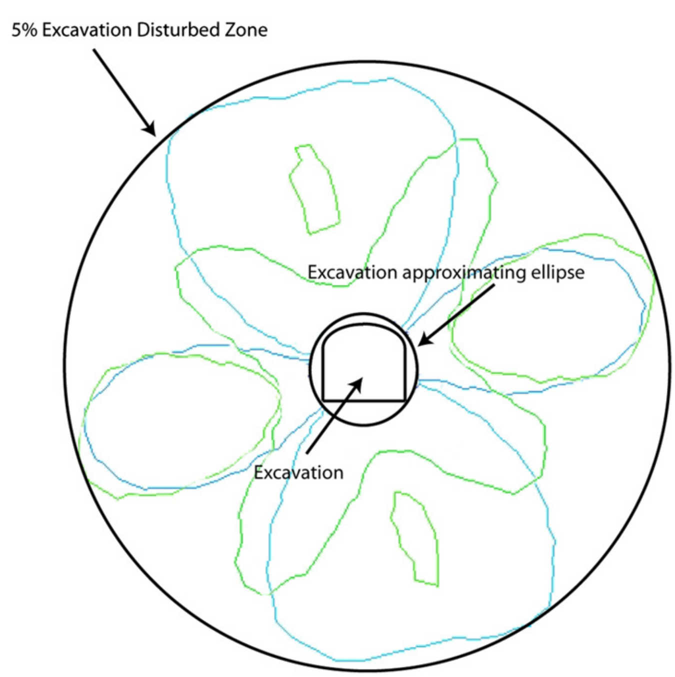

Fig. 1. Establishment of the EDZ for use in the mesh optimization algorithm.