International Journal of Rock Mechanics & Mining Sciences 48 (2011) 546–555

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of intact rocks

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ARTICLE INFO

Article history:
Received 10 February 2010
Received in revised form
4 January 2011
Accepted 9 February 2011
Available online 16 March 2011

Keywords:
Intact rock
Triaxial
Polyaxial
Strength
Criterion

ABSTRACT

Triaxial or polyaxial strength of rocks is required while analysing many civil and mining engineering structures in rocks. Mohr–Coulomb criterion is the most widely used strength criterion in rock engineering problems. In its present form the criterion suffers from two major limitations. Firstly, it represents the strength of rock as a linear function of confining pressure. Secondly, the effect of intermediate principal stress is not considered by this criterion. In the present study, this criterion is modified to take into account the non-linearity and effect of intermediate principal stress on strength behaviour. Barton's [1] critical state concept for rocks has been employed for this purpose. The applicability of the proposed simple non-linear triaxial and polyaxial strength criteria has been verified by applying them to experimental results for the intact isotropic rock material available from literature and comparing the prediction with the other popular criteria in vogue. The agreement has been found to be excellent. The applicability of the concept to jointed rocks will be discussed in separate publication.

1. Introduction

The rocks generally encountered in civil and mining engineering projects are subjected to in-situ stress fields. Excavation of an underground opening in such a rock redistributes the stresses and, failure of rock takes place under the influence of a complex multiaxial stress state. Strength criteria are used to define the strength of rocks subjected to given stress field and also to determine the extent of plastic zone if failure occurs. The strength criteria are, therefore, very important in designing structures in rocks.

A number of strength criteria have been proposed for intact rocks by various researchers in past. Many of them are based on sound principles of mechanics. But for practical applications, it is more important, how easily the parameters of a strength criterion can be obtained in the field. This is probably the reason that, despite having numerous theoretical strength criteria, the conventional Mohr–Coulomb strength criterion is the most popular and widely used strength criterion. The criterion however suffers from two major limitations: (a) it is a linear criterion and expresses the strength of the rock as a linear function of confining pressure or normal stress, and (b) in its present form, the criterion ignores the effect of intermediate principal stress σ_2. There is ample evidence available that, in general, the intermediate principal stress does have substantial influence on the strength of rocks [2–10], barring a few cases of non-dilatant rocks [9].

An attempt has been made in this paper to overcome the above-mentioned limitations. The non-linear response has been incorporated in the Mohr–Coulomb criterion. The shear strength parameters c and ϕ, obtained from triaxial strength tests performed at low confining pressure, are directly used in the modified criterion to determine the triaxial strength of the rock at higher confining pressure. Furthermore, the proposed criterion has been extended to determine strength under polyaxial stress conditions. The applicability of the criterion has been validated by applying it to extensive databases of triaxial and polyaxial stress strength tests from available literature. The applicability of the proposed strength criterion to jointed rocks and rock masses will be discussed in future publication.

2. Critical state concept for rocks

If a rock is tested under confined condition, its strength increases with increase in confining pressure. The rate of increase in strength is high at low confining pressure. As confining pressure is increased, the rate of increase in strength decreases. The instantaneous friction angle is, therefore, high at low confining pressure conditions. The reason behind this is the dilatant and brittle behaviour exhibited by rocks at low confining pressure. The micro-cracks, which exist in a rock, open up at the onset of the failure, due to which the volume of the rock increases at the

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1365-1609/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijrmms.2011.02.004

time of failure. This results in a higher friction angle at low confining pressure conditions. At higher confining pressure, the tendency of opening of micro-cracks and hence the dilation of the rock is suppressed. The failure mechanism shifts from brittle to ductile. At sufficiently high confining pressure the rock becomes completely ductile. On further increase in confining pressure the rock enters the critical state.

The failure envelope of rock plotted in $\tau-\sigma$ space is non-linear and concave downward (Fig. 1a). The tangential gradient of the envelope is steep where crossing the shear stress axis and tends to become asymptotic to a horizontal line at sufficiently high confining pressure. Barton [1] terms this phenomenon as the critical state of rocks. He states "*critical state for an initially intact rock is defined as the stress condition under which Mohr envelope of peak shear strength of the rocks reaches a point of zero gradient. This condition represents the maximum possible shear strength of the rock. For each rock, there will be a critical effective confining pressure above which the shear strength cannot be made to increase*". The above argument of Mohr envelope approaching zero gradient is also supported by Hoek [11] who analysed triaxial test data for Indiana limestone (Fig. 1b). It can be seen from Fig. 1b (redrawn from Hoek [11]) that the rock enters the ductile region at high confining pressure, and the instantaneous friction angle ϕ approaches zero. Fig. 1c (from [13]) also indicates that the critical state concept as suggested by Barton is applicable to rocks. A significant conclusion from these studies is that the strength response of rocks at higher confining pressure considerably deviates from linear response and the non-linearity should be given due importance while assessing rock strength at a given confining pressure.

3. Modified Mohr–Coulomb strength criterion

3.1. General

Let us start with the triaxial stress conditions to suggest a non-linear strength criterion. Fig. 2 shows a plot of Mohr–Coulomb linear criterion in $(\sigma_1-\sigma_3)$ vs. (σ_3) space. The Mohr–Coulomb linear criterion may be expressed in terms of σ_3 and σ_1 as

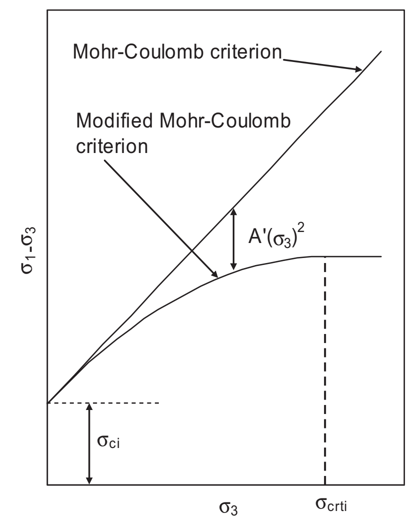

Fig. 2. Modified Mohr–Coulomb criterion.

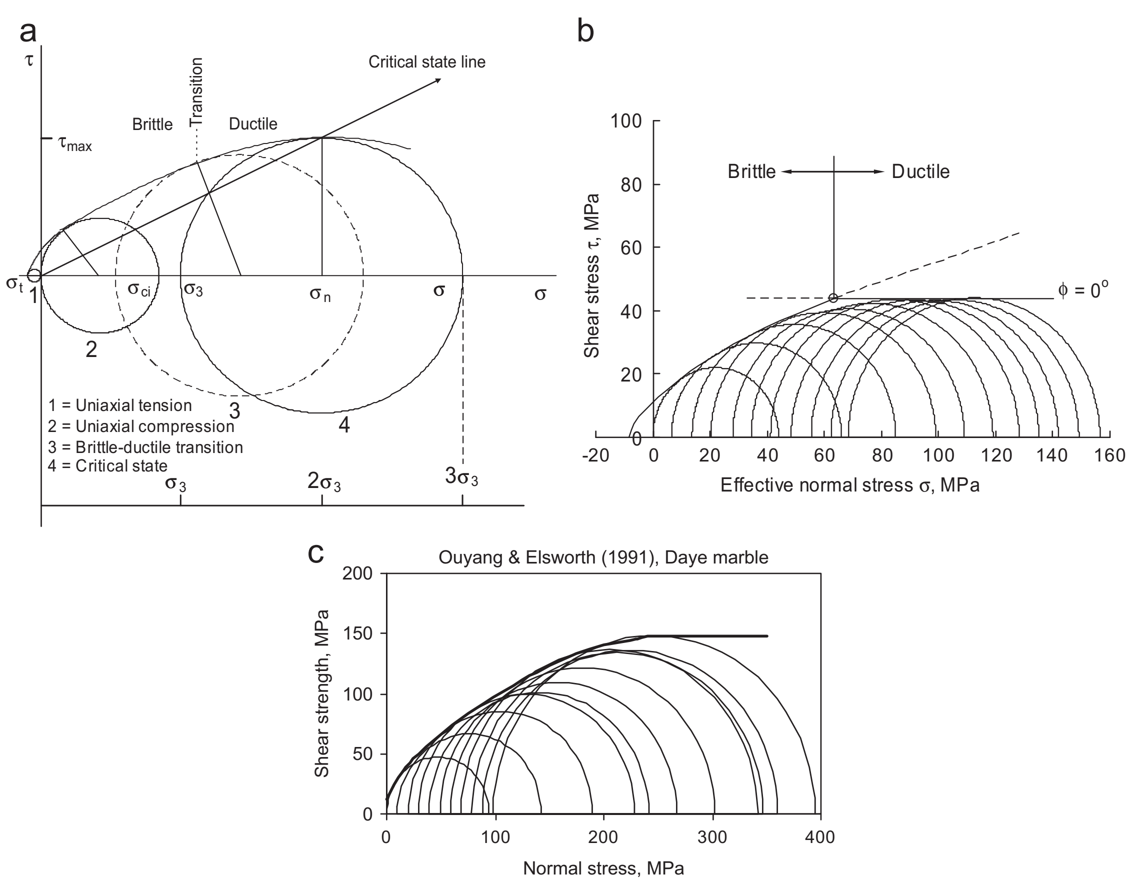

Fig. 1. (a) Critical state of intact rocks indicating Mohr envelope reaching a point of zero gradient [1]; (b) critical state for Indiana limestone ([12]; redrawn from [11]); and (c) critical state for Daye marble (redrawn from [13]).