International Journal of Rock Mechanics & Mining Sciences 48 (2011) 964–975

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Footwall slope stability analysis with the numerical manifold method

Y.J. Ning [a], X.M. An [a], G.W. Ma [b,*]

[a] School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798, Singapore
[b] School of Civil and Resource Engineering, University of Western Australia, 35 Stirling Highway, Crawley WA 6009, Australia

ARTICLE INFO

Article history:
Received 28 July 2010
Received in revised form
14 March 2011
Accepted 20 June 2011
Available online 2 July 2011

Keywords:
Numerical manifold method
Slope
Instability
Discontinuity modeling

ABSTRACT

This paper extends the numerical manifold method (NMM) for footwall slope stability analysis. A fracturing algorithm based on the Mohr–Coulomb criterion with a tensile cutoff is implemented into the NMM code. The developed program is first calibrated by simulating four typical crack problems. It is then applied to analyze the potential footwall slope instability. Parametric studies with respect to dip of a bedding plane, orientation of a predominant joint set, the bedding profile, etc., are carried out. Numerical results indicate that the developed program can simulate the opening and sliding along pre-existing discontinuities, fracturing through intact rock, as well as kinematics of the failed slope, and can also reproduce the major failure mechanisms observed in footwall slope collapses. The NMM is promising for such problems and deserves to be further developed to be practically used in natural/excavated rock slope stability analysis and open pit slope design.

1. Introduction

Open pit mining is a very cost-effective mining method. In open pit mining, mineral deposits are mined from the ground surface downward and thus pit slopes are formed. Since it is impossible to maintain stable vertical slopes, the pit slopes must thus be inclined to prevent failure. With the mining depths steadily increased to 1000 m or even higher during the last few decades, design of open pit slope angles is becoming more and more important. Small changes in the overall pit slope angle have significant influence on the overall economy of the mining operation. An ideal design method must be able to: (i) predict where and under what conditions a failure surface may develop; (ii) predict the kinematics of the failed slope. Current design methods can be distinguished into four groups: (1) empirical design methods; (2) physical model tests; (3) limit equilibrium methods; (4) numerical methods.

Empirical methods design slope angles using past slope behavior experiences. Various design charts [1,2] are available now. Although attractively simple, purely empirical design methods are not precise enough for the design of overall and final pit slopes in open pit mining.

Physical model tests provide the means of simulating the conditions of an actual slope in a controlled environment, where parameters can be easily varied and their effects on the slope stability can be studied. Some of the most interesting model tests with respect to possible failure mechanisms in large scale slopes are reported and discussed in [2]. Physical model tests have dramatically increased the knowledge and understanding of the possible failure modes in rock slopes. However, model tests are usually time-consuming and costly. Today, numerical methods have to some extent replaced physical model tests as a means of conducting sensitivity studies.

Based on limit equilibrium methods [3], a possible failure surface can be assumed for a slope and then the factor of safety is calculated accordingly. It is simple and has been widely used. Various methods are available, both analytically and graphically, which have been used successfully for rock slope design. However, these methods oversimplify each block as a rigid body. Moreover, the failure surface must to some extent be known beforehand. Degree of complexity in both problem geometry and material behavior, which can be handled, is limited [4]. The kinematics of the failed slope cannot be described.

Numerical methods have become very popular in recent years with the following various conveniences: (1) both stress and displacements in a block can be calculated; (2) various constitutive relations (e.g., anisotropic, plastic, etc.) can be employed; (3) complex slope geometries can be handled and parametric studies can be conducted; (4) groundwater flow can be coupled. Today, there is a vast number of different numerical methods available. Generally, they are categorized into two groups: continuum-based methods (e.g., the FDM, FEM, BEM, etc.) and discontinuum-based methods (e.g., the distinct element codes such as UDEC [5], 3DEC [6], etc., and the discontinuous deformation analysis (DDA) [7]). In continuum-based models, stresses are well resolved; however, the number of discontinuities that can be

* Corresponding author.
E-mail address: ma@civil.uwa.edu.au (G.W. Ma).

1365-1609/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijrmms.2011.06.011

handled is limited. In addition, it fails to model accurately the kinematics of collapsed slopes. In discontinuum-based methods, the discontinuities presented in the rock mass are modeled explicitly. Discontinuum-based codes have achieved great success in accounting for the kinematics of discrete blocks. They, however, have difficulty to simulate failures through intact rock. Either continuum- or discontinuum-based methods individually encounter difficulties in realistic simulations of rock slope failures.

In fact, rock slope failure is a typical combined continuum–discontinuum problem, and it is better to be simulated by a hybrid continuum–discontinuum method. Recently, a hybrid finite-/discrete element code ELFEN was developed and successfully applied to simulate rock slope failures [8,9]. In the ELFEN, two methods are proposed to insert the discrete crack. Intra-element fracturing inserts the discrete crack in the actual direction, but requires remeshing to achieve an acceptable element topology. In contrast, the inter-element fracturing snaps the discrete crack to the most favorably oriented existing element side, which does not require remeshing anymore; however, it needs very fine meshes to capture the fracture orientation accurately. The DDA with discrete blocks bounded by artificial joints was also used to model the failures in intact rock [10,11]. Fracturing is realized by converting artificial joints into real joints once a failure criterion is met; however, its predicted crack trajectory depends on the advance block discretization configuration. The particle flow code (PFC) by Itasca [12,13] with a capability of modeling both intact rock and joints, has shown its power in solving problems related to rock masses. For example, the PFC2D has been used to study on the stability of heavily jointed rock slopes [14].

The recently developed numerical manifold method (NMM) is another combined continuum–discontinuum method. The NMM was initially developed by Shi in [15,16] and later extended by various researchers for crack problems [17–20], heterogeneous solids [21–24], high-order formulations [25], and so on. A recent survey on the NMM can be found in [26]. The distinct features of the NMM are manifested in three aspects. First, the mesh used to generate the NMM model is totally free from the problem geometries, which makes the meshing task more convenient and remeshing totally avoided for discontinuity evolution. Second, discontinuities are modeled in a straightforward manner by its innovative dual cover system, without the need of *a prior* assumption or interface elements. Third, it integrates the FEM and DDA, thus makes it possible to model continuum, transition from continuum to discontinuum, as well as discontinuum in a unified framework.

In this paper, the NMM is extended to simulate the complete rock slope failure process, including opening and sliding along pre-existing discontinuities, fracturing through intact rock, and the final kinematics of failed slopes. We take the footwall slope for an example. Footwall slopes are often excavated parallel to dip and extensive along both strike and down-dip. When mining continues down-dip, the length of the exposed footwall slope increases and the potential instability of the slope may arise. Footwall slope instability is a relatively common occurrence in open pit mines situated in areas of steeply dipping strata particularly where mountainous topography exists [4]. The importance of footwall instability has long been recognized. The failure mechanisms of footwall slopes have been well documented in [27–29]. The footwall slope failures have been reproduced in [30] using the UDEC. In their studies, due to the constraints of the numerical tool, persistent discontinuities are assumed. Failures only occur along the pre-defined discontinuities. However, it is highly unlikely that such a network of fully persistent discontinuities exists in nature. Actual slopes involve non-persistent joints. Their failure includes both sliding along persistent discontinuities as well as fracturing through intact rock. In this paper, we will extend the NMM to account for the complete rock slope failure process. Potential modes of footwall slope instability will be investigated. Major failure mechanisms of footwall slopes will be reproduced to demonstrate the capacity of our developed NMM program.

2. Basic concepts of the NMM

The NMM uses a set of overlapped small patches to cover the problem domain Ω, as shown in Fig. 1. The edges of the small patches are not compulsory to coincide with neither the external boundary nor the internal discontinuities. Each small patch is termed as a mathematical cover (MC), denoted by M_I ($I=1-n_M$), where n_M is the number of MCs employed for a problem. External boundary and/or internal discontinuities may intersect each MC into several isolated pieces, then each piece (if it is within the problem domain) is termed as a physical cover (PC), denoted by P_I^j ($j=1-m_I$), where m_I is the number of PCs generated from M_I. For the example in Fig. 1, MC M_I is partially outside the problem domain, thus it forms one PC P_I^1 within its material portion; another MC M_J is cut by an internal crack into two isolated pieces and both are within the problem domain, thus it forms two PCs P_J^1 and P_J^2. Here, each PC has two indices, i.e., I and j. To simplify an implementation, we reallocate a single index to each PC, expressed as

$$P_i \triangleq P_I^j \tag{1}$$

where i is calculated by

$$i(I,j) = \sum_{l=1}^{I-1} m_l + j \tag{2}$$

The common area of several PCs is defined as a manifold element (ME).

On each MC M_I, a partition of unity (PU) φ_I (also termed as weight function in [15,16]), which satisfies

$$\begin{aligned} &0 \le \varphi_I \le 1, \forall \mathbf{x} \in M_I \\ &\varphi_I = 0, \forall \mathbf{x} \notin M_I \\ &\sum_I \varphi_I = 1, \forall \mathbf{x} \in \Omega \end{aligned} \tag{3}$$

is defined.

On each PC P_I^j (or P_i), a local approximation function (also known as cover function in [15,16]) $\mathbf{u}_I^j(\mathbf{x})$ (or $\mathbf{u}_i$), which reflects the local characteristic of the solution, is defined. The convenient choice for a basis of local approximation spaces is polynomial functions, which can approximate smooth functions well, e.g.,

$$S^c = \{1, x, y, \ldots, x^p, x^{p-1}y, \ldots, xy^{p-1}, y^p\} \tag{4}$$

for a two-dimensional problem, where the superscript 'c' stands for conventional. Within the PU framework, high-order polynomials up to degree p, $p \ge 1$, can be employed to improve the

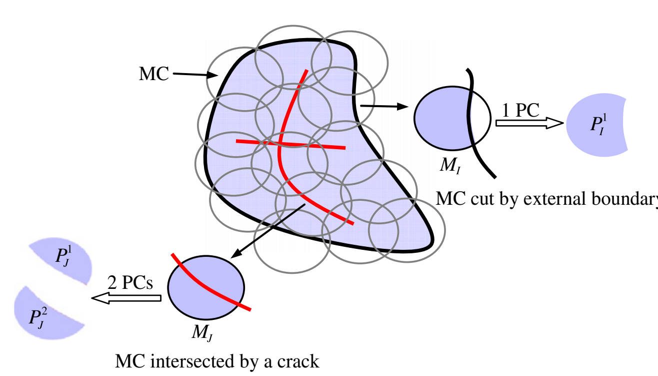

Fig. 1. Illustration of basic concepts of the NMM.