International Journal of Rock Mechanics & Mining Sciences 48 (2011) 476–481

Contents lists available at ScienceDirect

International Journal of
Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

In situ stress determination from inversion of hydraulic fracturing data

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ARTICLE INFO

Article history:
Received 27 June 2010
Received in revised form
11 August 2010
Accepted 28 August 2010
Available online 28 September 2010

Keywords:
Hydraulic fracture
In situ stress
Inversion
Inclined wellbores

ABSTRACT

The paper presents a new method, based on rigorous principles of mechanics, for determining the in-situ rock stress state based on hydraulic fracturing data. A solution can be obtained from a single data set which includes breakdown pressure, fracture angular position and trace angle. The inversion methodology is demonstrated on a case history from the Kuparuk River field, Alaska as reported by [1] and is shown to give good agreement with observed field data.

1. Introduction

Hydraulic fracturing is the best-known method of assessing the state of in situ stress at great depths. The technique originated in the 1940s as an oilfield stimulation technique designed to intensify production by fracturing a segment of a wellbore through pressurization; the fracture was then extended by additional pumping and maintained open (or propped) by injecting solid particles such as sand grains or glass beads. Following the great success of hydraulic fracturing as a stimulation technique, attempts were made to understand the mechanisms behind it. The most important effort in the interpretation of hydraulic fracturing mechanism was made by Hubbert and Willis [2] who used the theory of elasticity to reach the conclusion that the direction of the induced hydraulic fracture and the pressures recorded during borehole pressurization are directly related to the principal in situ stresses. Fairhurst [3] was among the first to advocate the use of hydraulic fracturing for in situ stress determination. Haimson and Fairhurst [4] extended Biot's theory of poro-elasticity to include both non-penetrating and penetrating injection fluid cases. Hydraulic fracturing has now become one of the key methods for in-situ rock stress estimation as suggested by the International Society for Rock Mechanics (ISRM) [5]. A detailed history of the method and a thorough description of the equipment, setup, test data interpretation and in situ stress derivation are presented in Ref. [6].

Although hydraulic fracturing is commonly conducted in vertical holes, testing of inclined or deviated wellbores is often required. For example, in the petroleum industry inclined holes are increasingly used for added efficiency in production. In geotechnical site investigations, exploratory inclined holes are often drilled in order to intersect vertical joint sets which could be missed by vertical holes. Additionally, rock properties or drilling difficulties may cause unintended deviated holes.

A technique for inverting results from a minimum of two leak-off tests at different well inclinations and azimuths was presented in Refs. [7,8]. This method gives an estimate of both horizontal stress magnitudes and directions. However, the published technique suffers from the assumption that shear stresses are neglected. Gjønes et al. [9] present results based on a model that also includes the shear stresses. Djurhuus and Aadnoy [10] present a theory for determining the in situ stress state from multiple fracturing data and induced fractures from image logs. A solution can be obtained with a minimum of two data sets. However, using an inversion technique, a solution can be obtained with any number of data sets, as the solution is over determined.

A fundamental understanding of fracture initiation in arbitrarily inclined wellbores under various in situ initial stress conditions is essential for the efficient and effective design of hydraulic fracture systems. The basic equations describing the stress distribution around a horizontal, vertical and inclined wellbore may be derived from the solutions developed by Kirsch [11], Fairhurst [12] and Bradley [13], respectively. It is general believed that a fracture initiates when the maximum tensile stress induced at any point around the wellbore exceeds the tensile strength of the formation at that point. When this occurs, the resulting fracture on the wellbore wall will have an

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doi:10.1016/j.ijrmms.2010.08.018

orientation that is perpendicular to the direction of the most tensile principal stress. The angle between the wellbore generatrix and the fracture orientation on the wellbore wall is called the trace angle (e.g., [14–17]), which can be observed by high-resolution electrical imaging technologies (e.g., [18]). Some previous attempts have been made to use the fracture trace angle to determine in situ stresses (e.g. [10,16]). This paper presents a new method for determining the in situ stress state from hydraulic fracturing data. It is shown that an analytical solution can be obtained when fracture trace angle is available. Later in the paper, this new method is applied to data from the Kuparuk River field in Alaska as reported in Ref. [1], and shown to predict in-situ stresses in good agreement with field data. When fracture trace angles are not available, other incomplete inversion methods can be used, such as "leak-off" data from the North Sea (e.g. [9,10]).

2. Theoretical development for stresses and hydraulic fracturing

Hydraulic fracturing consists of sealing off a short segment (typically 0.5–2 m) of a wellbore or borehole at the desired depth, injecting fluid (usually water) into it at a rate sufficient to raise the hydraulic pressure quite rapidly (typically 0.1–1.0 MPa/s), and bringing about hydraulic fracturing. The latter is achieved when the borehole fluid pressure reaches a critical level called breakdown pressure. At breakdown the rock fractures in tension causing borehole fluid loss and hence a drop in pressure. When pumping is stopped, the hydraulic line to the testing interval remains in place. Following fracture, the pressure immediately decays, at first very quickly as the fluid chases the still extending fracturing tip, and then more slowly as the fracture closes, after which the only remaining fluid loss is due to seepage into the rock through the borehole wall. The "shut-in pressure" occurs at the transition between the fast and slow pressure decay and signifies the closure of the fracture.

The in situ principal stresses are assumed to be vertical and horizontal. The rock is assumed to be isotropic, homogeneous and linearly elastic. The ambient pore pressure in the rock is assumed to remain constant during the test. However, the method presented can easily be extended to cases where fluid penetrates into surrounding rock, in which case poro-elastic theory must be adopted.

Referring to Fig. 1 and assuming a compression positive convention, let σ_v, σ_H and σ_h ($\sigma_H > \sigma_h$) be the initial in situ vertical and horizontal principal stresses (σ_H is the most compressive horizontal principal stress). Let the origin of the principal stress axes lie at the center of the top of the inclined

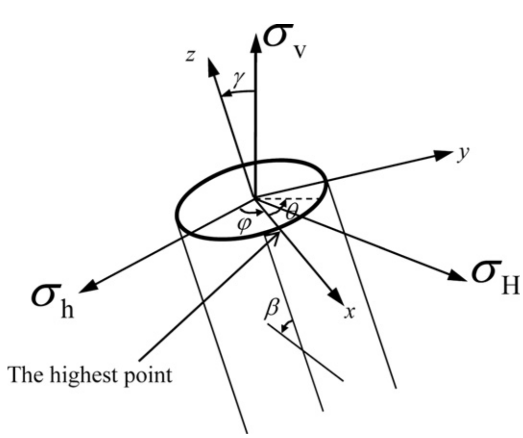

Fig. 1. Wellbore configuration.

wellbore shown in the figure. Consider the local coordinates of the wellbore (x, y, z) with the same origin, where the x-axis passes through the highest point of the circumference and the z-axis passes down the longitudinal axis. The wellbore azimuth φ is the horizontal angle between the vertical plane containing the σ_h-axis and the vertical plane containing the x-axis, measured counterclockwise as viewed down the σ_v-axis looking towards the origin. The wellbore inclination γ is the angle between the σ_v-axis and the z-axis measured clockwise as viewed down the y-axis looking towards the origin.

The geometry of fractures initiated along an arbitrarily inclined wellbore is strongly dependent on in situ stresses (σ_v, σ_H and σ_h), wellbore azimuth (φ) and inclination (γ). The fracture trace angle β between the fracture trace and wellbore generatrix is measured in a clockwise direction when looking outwards from the wellbore axis.

For an arbitrarily oriented wellbore, the rotation of the stress tensor from the in situ coordinate system to a local wellbore coordinate system (Fig. 1) is given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \cos^2\varphi\cos^2\gamma & \sin^2\varphi\cos^2\gamma & \sin^2\gamma \\ \sin^2\varphi & \cos^2\varphi & 0 \\ \cos^2\varphi\sin^2\gamma & \sin^2\varphi\sin^2\gamma & \cos^2\gamma \\ -\sin\varphi\cos\varphi\cos\gamma & \sin\varphi\cos\varphi\cos\gamma & 0 \\ \sin\varphi\cos\varphi\sin\gamma & -\sin\varphi\cos\varphi\sin\gamma & 0 \\ -\cos^2\varphi\sin\gamma\cos\gamma & -\sin^2\varphi\sin\gamma\cos\gamma & \sin\gamma\cos\gamma \end{bmatrix} \begin{Bmatrix} \sigma_h \\ \sigma_H \\ \sigma_v \end{Bmatrix}. \tag{1}$$

Let θ be the angle measured counterclockwise starting at $\theta=0$ on the x-axis as viewed down the z-axis looking towards the origin. Stresses at the wellbore wall are (e.g. [14,19])

$$\begin{aligned} &\sigma_r = P_w \\ &\sigma_\theta = \sigma_x + \sigma_y - 2(\sigma_x - \sigma_y)\cos(2\theta) - 4\tau_{xy}\sin(2\theta) - P_w \\ &\sigma_\zeta = \sigma_z - \upsilon\{2(\sigma_x - \sigma_y)\cos(2\theta) + 4\tau_{xy}\sin(2\theta)\} \\ &\tau_{r\theta} = 0 \\ &\tau_{\theta\zeta} = -2\tau_{xz}\sin\theta + 2\tau_{yz}\cos\theta \\ &\tau_{\zeta r} = 0 \end{aligned} \tag{2}$$

where P_w is the compressive applied wellbore pressure and υ is Poisson's ratio. Thus, the effective stress tensor is

$$\sigma' = \begin{bmatrix} \sigma'_r & 0 & 0 \\ 0 & \sigma'_\theta & \tau_{\theta\zeta} \\ 0 & \tau_{\theta\zeta} & \sigma'_\zeta \end{bmatrix} \tag{3}$$

where $\sigma'_r = \sigma_r - P_0$, $\sigma'_\theta = \sigma_\theta - P_0$, $\sigma'_\zeta = \sigma_\zeta - P_0$ and P_0 is the pore pressure.

The effective principal stresses at the wellbore can be found as the eigenvalues of the effective stress tensor, thus

$$\begin{aligned} &\sigma'_1 = \sigma'_r \\ &\sigma'_2 = \frac{(\sigma'_\theta + \sigma'_\zeta) + \sqrt{(\sigma'_\theta - \sigma'_\zeta)^2 + 4\tau_{\theta\zeta}^2}}{2} \\ &\sigma'_3 = \frac{(\sigma'_\theta + \sigma'_\zeta) - \sqrt{(\sigma'_\theta - \sigma'_\zeta)^2 + 4\tau_{\theta\zeta}^2}}{2} \end{aligned} \tag{4}$$

with corresponding eigenvectors

$$\vec{\sigma}'_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$