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Estimation of the tensile elastic modulus using Brazilian disc by applying diametrically opposed concentrated loads

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ABSTRACT

The tensile elastic modulus E_t of a rock is different from the compressive elastic modulus E_c, due to inhomogeneity and microcracks. There is no convenient method to obtain E_t except using direct tension tests. However, the direct tension test for rock materials is difficult to perform, because of stress concentrations, and the difficulty of preparing specimens. We have developed a new method to determine E_t of rock materials easily and conveniently. Two strain gauges are pasted at the center part of a Brazilian disc's two side faces along the direction perpendicular to the line load to record tensile strain, and a force sensor is used to record the force applied; then the stress–strain curve can be obtained; finally the E_t can be calculated according to those related formulas which are derived on the basis of elasticity theory. Our experimental results for marble, sandstone, limestone and granite indicate that E_t is less than E_c, and their ratio is generally between 0.6 and 0.9.

1. Introduction

In general, rocks are inhomogeneous, and contain numerous microcracks. Consequently, rocks show different behavior under tensile and compressive conditions. Accordingly, there are two kinds of elastic modulus: the compressive elastic modulus E_c and the tensile elastic modulus E_t. Parameters such as Young's modulus and Poisson's ratio are expected to be different under compressive or tensile stress [1,2]. The data reported by Krech et al. [3] and Liao et al. [4] indicate such differences for Young's modulus and Poisson's ratio in some rock types (granite, quartzite, sandstone, limestone, argillite).

The modulus E_c and the compressive strength σ_c are easy to measure in the laboratory by uniaxial compression tests. But the parameter E_t and the tensile strength σ_t are difficult to obtain by direct tension tests, because it is very difficult and complicated to prepare test specimens, and it is easy to generate stress concentrations at the ends of the specimen. In order to solve this problem, the International Society for Rock Mechanics (ISRM) officially proposed the Brazilian test as a suggested method for determining the tensile strength σ_t of rock materials [5]; however, there is no indirect test proposed by the ISRM to determine the tensile elastic modulus E_t. In this study, a convenient and maneuverable method for determining E_t of rock materials with the Brazilian disc approximately is to be developed.

In the Brazilian test, a disc specimen is compressed with diametrically opposite and symmetric line loads [6,7]. The theoretical basis for the Brazilian test is the analytical solutions that have been obtained by many researches for isotropic or transverse isotropic materials under concentrated loads, loads that are distributed over a small arc of the disc's circumference [8–11]. Fairhurst [12] discussed the validity of the Brazilian test, and concluded that failure is expected to initiate at the center of disc, but actually the failure sometimes initiates at the loading points. Hudson [13] verified this conclusion with experiments. Guo et al. [14] developed a simple method to measure the opening mode (mode-I) fracture toughness K_{Ic} with the Brazilian disc. Wang et al. [15] later made some improvement to Guo's method for determining K_{Ic}.

Much attention has been placed on the elastic modulus by researchers. Hondros [8] developed an approach to measure the elastic modulus E and Poisson's ratio ν with the Brazilian disc. He also gave a complete stress analytic solution for the case of a radial load distributed over a finite circular arc of the disc. However, this kind of loading is very difficult to obtain in the laboratory. Consequently, there will be some differences between the actual stress field and the ideal analytical solution. And there is another problem for the method: the theory of the method is based on the strain of the center of the disc, but the strain measured by the strain gauge is the contribution of a line segment

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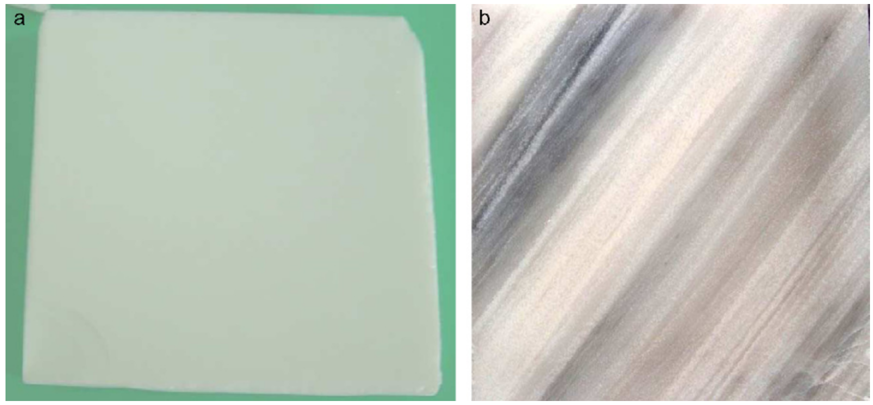

Fig. 1. (a) is white marble which seem to be homogenous and isotropic and (b) banded marble shows obvious transverse isotropy.

near the center of the disc. Yu et al. [16] invented a method for determining E with the Brazilian test proposed by ISRM. They recorded the force applied and the displacement of loading point in experiment, and then a force–displacement curve could be obtained. The slope of the line section of force–displacement curve was defined as E_d. The elastic modulus E could be determined by E_d multiplying a correction coefficient k. According to finite-element analysis (ANSYS) and experiment, using Three Gorges granite, they concluded that k is about 19.2. This method was an improvement, but it was just based on linear elastic, isotropic finite-element analysis and regressing, fitting test data. Its credibility and reliability may be low for other rock types. Wang et al. [17–19] developed a similar method for determining E with flattened Brazilian disc. They gave out an approximate formula to calculate the relative displacement between the two ends according to Cauwellaert's result for a uniformly and parallelly distributed load applied on a section of a circular arc [20]. However, You and Su [21] disputed Wang's method. Because the loading and geometry between flatted Brazilian disc and Cauwellaert's complete disc were completely different, they concluded that it was incorrect to use Cauwellaert's results.

The elastic modulus E mentioned above is the compressive elastic modulus. Some attention has also been placed on the tensile elastic modulus. Li and Yin [22] used pure bending beam to measure the E_c and E_t. Two strain gauges were pasted on the upper surface (compressive zone) and lower surface (tensile zone), to record the compressive and tensile strain, respectively. The strongpoint of this method is that it can obtain E_c and E_t at the same time. Zhang et al. [23] devised a method to calculate the tensile elastic modulus E_t of rock with a cracked Brazilian disc. A small vertical, straight and through notch was required at the center of disc. However, the theory of this method is immature at present, and it is difficult to make the notch required in Brazilian disc.

As we know, some rock types show non-homogeneity to some extent, and contain many microcracks. These two factors lead to some anisotropy for rock materials. Furthermore, the level of microcracking and the orientation of microcracks also have great effect on the mechanical properties of rock materials. However, it is very difficult to study the effect quantitatively at present. Therefore, for the sake of simplicity, and also adopting the same model proposed by ISRM for the Brazilian test who treat the rock material as an equivalent isotropic continuum medium, we also consider the disc to be an equivalent isotropic continuum medium. Correspondingly, in fact the elastic modulus and Poisson's ratio are the equivalent ones. Of cause, the model adopted here is not suitable for those rock types that show anisotropy or transverse isotropy. For example, Fig. 1a is a white marble, which seems to be homogenous and isotropic. Fig. 1b is a banded marble, which shows transverse isotropy. Certainly, the white marble is suitable for the model proposed here, whereas the banding marble is not suitable.

In this study, a simple and convenient test method is proposed for determining E_t for isotropic rock materials with the Brazilian disc. The configuration of the test is shown in Fig. 2. The core idea of the proposed method that two strain gauges are pasted, respectively, at the center of disc on the both side faces along the direction perpendicular to the line load P (Fig. 3) to record the tensile deformation of the center part. Then, according to the stress obtained through elasticity theory and the recorded strain, the equivalent tensile elastic modulus E_t can be calculated. Obviously, the loading manner of the test method proposed here is completely different from that proposed by ISRM. The ISRM suggests that two concave loading plates can be used to apply load in order to distribute the load along an arc of the disc. The reasons that the test configuration shown in Fig. 2 is adopted in this study are as following. On the one hand, the stress analytic solution obtained by Hondros [8] for a pair of distributed loads applied over an arc of the disc oppositely and diametrically (Fig. 4) based on isotropy is

$$\sigma_r = -\frac{2p}{\pi}\left\{\alpha + \sum_{n=1}^{n=\infty}\left[1-\left(1-\frac{1}{n}\right)\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)^{2n-2}\sin 2n\alpha\cos 2n\theta\right\}$$
$$\sigma_\theta = -\frac{2p}{\pi}\left\{\alpha - \sum_{n=1}^{n=\infty}\left[1-\left(1+\frac{1}{n}\right)\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)^{2n-2}\sin 2n\alpha\cos 2n\theta\right\}$$
$$\tau_{r\theta} = -\frac{2p}{\pi}\left\{\sum_{n=1}^{n=\infty}\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)^{2n-2}\sin 2n\alpha\cos 2n\theta\right\} \quad (1)$$

where p is the applied pressure, R is the radius of disc, r and θ are the polar coordinates of a point in disc, and α is the half central angle related to the distributed load applied. From the equation above, we know that the stress field of the disc subjected to a pair of distributed load is the function of α. Obviously, the magnitude of α affects the stress distribution in the disc directly. According to ISRM's suggestion [5], if the standard concave loading plates are used in the Brazilian test, the 2α is about 10° at failure. As far as we know, the disc would show some plastic properties, rather than complete elasticity when approaching failure. However, the parameter of tensile elastic modulus E_t is a physical quantity of the elastic stage in tension of rock materials. Therefore, 2α is certainly not 10°, and there is no way to exactly know the specific value of 2α when the disc is in the elastic stage. Furthermore, 2α is a variable in the processing of loading. Additionally, the value of