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Technical note

Fully coupled dual-porosity model for anisotropic formations

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1. Introduction

In recent decades, all petroleum reservoir problems involve two basic elements: fluid and rock. We are interested in two particular processes associated with them: fluid flow and geo-mechanics. Fluid flow is essential in a petroleum reservoir study. Geo-mechanics is believed to be important in several petroleum-engineering activities such as drilling, borehole stability, hydraulic fracturing and production-induced compaction and subsidence.

As we know, the production from naturally fractured reservoirs has a great potential worldwide, and many profitable reservoirs are naturally fractured. For naturally fractured reservoirs, economical petroleum production relies on the fracture permeability. The natural fractures basically are the product of the evolving rock stress state. Therefore, any disturbance of the stress field, such as due to fluid production/injection, can affect the existing fractures and the associated reservoir performance. A coupled fluid-flow/ geo-mechanics model can provide a rational tool for a better understanding and management of naturally fractured reservoirs.

Naturally fractured reservoirs are often modeled by the dual-porosity type of concept developed by Barenblatt et al. [1] and Warren and Root [2]. Models incorporating both Biot's poroelastic theory [3] and Barrenblatt's dual-porosity concept have been studied by several authors. The models they have established can be classified into two types, based on the approach taken.

One approach is based on the mixture theory, and was adopted by Wilson and Aifantis [4], Beskos and Aifantis [5], Bai et al. [6], and Berryman and Wang [7]. The resulting formulations have two related features: the first is that all the fluid-flow equations in a "mixture" have the same functional form as that of a single-porosity if the fluid exchange term is dropped; and the second is that phenomenological coefficients are proposed first, and their physical interpretations are deduced after the completion of the formulation. The first point implies that the stress-dependent rock properties in one continuum are independent of the other mixing continua. This in turn may cause difficulty for the later physical interpretation (i.e., the second one) and even inconsistency with the geo-mechanical equations adopted.

The other approach follows the route of conventional fluid-flow modeling. Coupling of geo-mechanics is identified through stress-dependent rock properties and during the development process. Interpretation of the stress-dependent properties is therefore critical to achieving a proper coupling. This approach was adopted by Duguid and Lee [8], Valliappan and Khalili-Naghadeh [9], Khalili-Naghadeh and Valliappan [10], Chen et al. [11], and Li et al. [12].

In the model of Duguid and Lee [8], an incompressible solid was assumed. Also, no explicit rock compressibilities (solid, pore, and bulk) appear in their fluid-flow equations. Explicit rock compressibilities were considered by Valliappan and Khalili-Naghadeh [9] (also [10]). But except for the case of an incompressible solid, their two fluid pressure equations do not collapse to the corresponding single-porosity equation when the two fluid pressures reach equilibrium (i.e., $p_1 = p_2$). So it suggests that incompressible solid phase had been implicitly adopted in their general derivations which, however, contradicts their intention and the presented equations.

The disadvantages identified in the above models are resolved in the model of Li et al. [12]. Specifically, (i) internal model consistency is maintained, and

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(ii) continuity between the single-porosity and dual-porosity systems can be established smoothly.

The above models are all based on the assumption of isotropy, whereas most rocks are characterized by anisotropy of various degrees. Therefore, research on the behavior of anisotropic dual-porosity media is of relevance to oil and gas production. This paper is focused on the development of the governing equations for a fully coupled dual-porosity model for anisotropic rock formations.

2. Geo-mechanical model

The three basic principles of poroelastic theory are: stress equilibrium, strain–displacement, and strain–stress-pressure relations. Mathematically, these are the static-equilibrium equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \tag{1}$$

the strain–displacement relation:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

and the effective stress principle and the stress–strain constitutive relation for anisotropic double porous media (strain–stress-pressure relations):

$$\sigma_{ij}^{e} = \sigma_{ij} - \alpha_{ij}^{1} p_1 - \alpha_{ij}^{2} p_2 = C_{ijkl}\varepsilon_{kl}. \tag{3}$$

In Eqs. (1)–(3), σ_{ij} and ε_{kl} are the components of total stress tensor and bulk strain tensor, respectively. We should note that the sign convention here follows that in which compressive stresses are positive. Additionally, u_i is the solid displacement vector, C_{ijkl} denotes the elastic moduli tensor, σ_{ij}^{e} denotes the effective stress tensor, p_n is the nth fluid pressure, α_{ij}^{n} is the nth Biot coefficient tensor (where the subscript $n = 1, 2$ indicates the matrix-block and fractures, respectively). For anisotropic double porous media, α_{ij}^{n} has the following form [13]:

$$\begin{aligned} \alpha_{ij}^{1} &= \delta_{ij} - C_{ijkl} M_{klmn}^{*}, \\ \alpha_{ij}^{2} &= C_{ijkl} M_{klmn}^{*} - C_{ijkl} M_{klmn}^{s}, \end{aligned} \tag{4}$$

where δ_{ij} is Kronecker's delta ($\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ for $i \neq j$), M_{klmn} is the elastic compliance tensor, the superscript $*$ stands for a porous media of without fractures, and the subscripts s represents solid. Body forces and inertial effects are neglected in Eq. (1). Small strains are implied in Eq. (2). Substituting Eq. (3) into the equilibrium Eq. (1) gives

$$\frac{1}{2} C_{ijkl}\left(\frac{\partial^2 u_k}{\partial x_j \partial x_l} + \frac{\partial^2 u_l}{\partial x_j \partial x_k}\right) = -\alpha_{ij}^{1}\frac{\partial p_1}{\partial x_j} - \alpha_{ij}^{2}\frac{\partial p_2}{\partial x_j}. \tag{5}$$

Eq. (5) gives the geo-mechanical model for anisotropic double porous media. Three simplifications of the geo-mechanical model are discussed next.

2.1. Transverse isotropy

The geo-mechanical model Eq. (5) becomes more tractable in the case of transverse isotropy. This is an important type of anisotropy in geophysical applications, since material properties are frequently isotropic in the bedding plane but differ in the direction normal to this plane. The anisotropy may be either structural (anisotropic pore geometry), intrinsic (anisotropic solid material), or both.

The tensor of elastic moduli for a transversely isotropic material has the form [14]:

$$\begin{aligned} C_{ijkl} = {} & \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\delta_{ij}\delta_{kl} + \alpha(\delta_{ik}h_j h_l + \delta_{jl}h_i h_k \\ & + \delta_{il}h_j h_k + \delta_{jk}h_i h_l) + \beta(\delta_{ij}h_k h_l + \delta_{kl}h_i h_j) + \gamma h_i h_j h_k h_l, \end{aligned} \tag{6}$$

where α, β, γ, λ, and μ are constants, and $h_i$is the directional cosine of symmetry axis. For isotropic materials, α, $\beta$and γ vanish; λ and μ are the Lamé constants. Introducing Eq. (6) into Eq. (5) give

$$\begin{aligned} & \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \alpha h_i h_k \frac{\partial^2 u_k}{\partial x_j \partial x_j} + (\mu + \lambda)\frac{\partial^2 u_j}{\partial x_j x_i} \\ & + (\alpha + \beta)h_i h_j \frac{\partial^2 u_k}{\partial x_j \partial x_k} + \alpha h_j h_k \frac{\partial^2 u_i}{\partial x_j \partial x_k} + (\alpha + \beta)h_j h_k \frac{\partial^2 u_k}{\partial x_j \partial x_i} \\ & + \gamma h_i h_j h_k h_l \frac{\partial^2 u_k}{\partial x_j \partial x_l} = -\alpha_{ij}^{1}\frac{\partial p_1}{\partial x_j} - \alpha_{ij}^{2}\frac{\partial p_2}{\partial x_j}, \end{aligned} \tag{7}$$

where the Biot coefficient tensors for a transversely isotropic material have the forms [13]

$$\begin{aligned} \alpha_{ij}^{1} &= \delta_{ij}(1 - AA_1^{*} - CB_1^{*}) - (BA_1^{*} + DB_1^{*})h_i h_j, \\ \alpha_{ij}^{2} &= \delta_{ij}(AA_1^{*} + CB_1^{*} - AA_1^{s} - CB_1^{s}) \\ & \quad + (BA_1^{*} + DB_1^{*} - BA_1^{s} - DB_1^{s})h_i h_j \end{aligned} \tag{8}$$

and

$$\begin{aligned} & A = 2\mu + 3\lambda + \beta, \quad B = 4\alpha + 3\beta + \gamma, \\ & C = \lambda + \beta, \quad D = 2\mu + 4\alpha + \beta + \gamma, \\ & A_1^{s} = 2\mu_1^{s} + 3\lambda_1^{s} + \beta_1^{s}, \quad B_1^{s} = 4\alpha_1^{s} + 3\beta_1^{s} + \gamma_1^{s}, \\ & A_1^{*} = 2\mu_1^{*} + 3\lambda_1^{*} + \beta_1^{*}, \quad B_1^{*} = 4\alpha_1^{*} + 3\beta_1^{*} + \gamma_1^{*}. \end{aligned}$$

The compliance tensor has the similar format as the above elastic modulus tensor, with compliance coefficients $\mu_1, \lambda_1, \alpha_1, \beta_1$and γ_1, which can be expressed in terms of the moduli $\mu, \lambda, \alpha, \beta$and γ.

2.2. Structural anisotropy

The geo-mechanical model described by Eq. (7) is simplified considerably when the anisotropy is structural rather than intrinsic, i.e., in the case of an isotropic solid material with an anisotropic pore structure. If the anisotropy is purely structural, so that the solid material is isotropic, then the Biot coefficient tensors Eq. (8)