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ScienceDirect
International Journal of Rock Mechanics & Mining Sciences 45 (2008) 1176–1186
International Journal of Rock Mechanics and Mining Sciences
www.elsevier.com/locate/ijrmms

Technical Note

A quantitative comparison of six rock failure criteria

Thomas Benz[a,*], Radu Schwab[b]

[a]Institute of Geotechnical Engineering, Universität Stuttgart, Pfaffenwaldring 35, 70569 Stuttgart, Germany
[b]Federal Waterways Engineering and Research Institute, Kussmaulstr. 17, 76187 Karlsruhe, Germany

Received 8 May 2007; received in revised form 8 January 2008; accepted 17 January 2008
Available online 1 April 2008

1. Introduction

This paper presents a quantitative comparison of some of the most common criteria for rock failure: The Mohr–Coulomb (MC) criterion, the Lade–Duncan (Lade) criterion, an approximation to the Wiebols and Cook (WC) criterion, the Hoek–Brown (HB) criterion, and the Mogi criterion. Furthermore, the new Hoek–Brown–Matsuoka–Nakai (HBMN) failure criterion proposed by Benz et al. [1] is considered in the comparison, which is conducted using true triaxial test data for eight different rocks. Of the several possible versions of the Mogi criterion, only its linear form, where its parameters are closely related to the MC shear strength parameters [2], is considered here. The HB criterion is employed in its generalized 2002 version [3]. The HB and the HBMN criteria use the same set of four material parameters. Two of them define intact rock behavior and two define rock mass behavior. Therefore, all criteria considered in this study require two material parameters to describe triaxial test data of intact rock.

In stress space, the six failure criteria differ in two main aspects. In $p-q$ space, the MC, ML, WC, and the Mogi criteria are linear, whereas the HB and the HBMN are non-linear. In contrast to the ML, WC, Mogi, and the HBMN criteria, the MC and the HB criteria are not influenced by the intermediate principal stress. Definitions of the stress invariants p and q are given in the next section.

The quantitative comparison of the six failure criteria follows the methodology introduced in Colmenares and Zoback [4], which is outlined in more detail in Section 3: A grid search was performed to determine the material parameters which result in the smallest misfit for each rock type. Conclusions are then drawn from the magnitude of misfit as well as from the material parameters that optimize each criterion's data fitting behavior.

2. Six rock failure criteria

2.1. Definition of stress measures

Within this paper, compressive stress is considered positive. Tensile stress is negative. Stresses are always considered to be effective values without any special indication by a prime. Principal stresses are denoted by only one subscript, e.g. σ_i with $i = 1, 2, 3$. The Roscoe stress invariants p (mean stress) and q (deviatoric stress), are defined as:

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad \text{and}$$

$$q = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}. \tag{1}$$

The Roscoe stress invariants relate to the octahedral normal stress, σ_{oct} and to the octahedral shear stress, τ_{oct} as:

$$\sigma_{oct} = p \quad \text{and} \quad \tau_{oct} = \frac{\sqrt{2}}{3} q, \tag{2}$$

respectively.

2.2. The Mohr–Coulomb criterion

The Mohr–Coulomb (MC) failure criterion [5] is one of the earliest and most trusted failure criteria for soils and rocks. Failure is assumed when in any (failure) plane the shear stress τ reaches the failure shear stress τ_{max} which is given by a functional relation of the form:

$$\tau_{max} = c + \sigma_n \tan \varphi, \tag{3}$$

*Corresponding author. Tel.: +49 711 685 69020;
fax: +49 711 685 62439.
E-mail address: Thomas.Benz@igs.uni-stuttgart.de (T. Benz).

1365-1609/$ - see front matter
doi:10.1016/j.ijrmms.2008.01.007

where c is the cohesion of the material, φ is the friction angle of the material, and σ_n is the normal stress acting on the respective failure plane. Alternatively, the MC criterion can be expressed in principal stresses as follows:

$$\sigma_1 - \sigma_3 = 2c\cos\varphi + \sin\varphi(\sigma_1 + \sigma_3). \tag{4}$$

The unconfined compressive strength σ_{ci} postulated by the MC criterion is then obtained by setting $\sigma_3 = 0$:

$$\sigma_{ci} = 2c\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right). \tag{5}$$

With Eq. (5), the MC criterion (Eq. (4)) can be also rewritten as:

$$\sigma_1 = \sigma_{ci} + \tan^2\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\sigma_3. \tag{6}$$

2.3. The modified Lade–Duncan criterion

Lade and Duncan [6] proposed the following failure criterion:

$$\frac{I_1^3}{I_3} = K_\varphi, \tag{7}$$

where $I_1 = (\sigma_1 + \sigma_2 + \sigma_3)/3 = p$ and $I_3 = \sigma_1\sigma_2\sigma_3$ are the first and third stress invariants, respectively, and K_φ is a material parameter. The original Lade–Duncan criterion does not take cohesion into account. As cohesion is an essential concept for rock failure criteria that are linear in $p-q$ space, a shift of the Lade–Duncan yield surface along the hydrostatic axis $p =$ const. was later introduced by Ewy [7]. The modified Lade–Duncan (Lade) criterion according to Ewy can be written as:

$$\frac{I_1^3}{I_3} = 3^3(3^3 + \eta), \tag{8}$$

where the stress in calculating the first and third stress invariants is shifted by $a = c/\tan\varphi$:

$$I_1 = ((\sigma_1 + a) + (\sigma_2 + a) + (\sigma_3 + a)) \tag{9}$$

and

$$I_3 = (\sigma_1 + a)(\sigma_2 + a)(\sigma_3 + a), \tag{10}$$

where c is the cohesion of the material and φ is the friction angle of the material. By means of Eq. (5), these modified stress invariants could likewise be expressed in terms of the unconfined compressive strength σ_{ci}. In triaxial compression, consistent results from the Lade criterion and the MC criterion are obtained by defining η as follows:

$$\eta = \frac{4\tan^2\varphi(9 - 7\sin\varphi)}{1 - \sin\varphi}. \tag{11}$$

2.4. The modified Wiebols and Cook criterion

Wiebols and Cook [8] derived a failure criterion by calculating the shear strain energy associated with micro-cracks in rock material. This model requires the knowledge of the coefficient of sliding friction between crack surfaces, the number of mean cracks per unit solid angle in a unit volume, and the critical value of shear strain energy. As the criterion cannot be evaluated in closed form in the case of polyaxial stresses, Zhou [9] proposed a similar but simpler criterion. The latter has been named the modified Wiebols and Cook (WC) criterion [4] and is explained in more detail here. The modified Wiebols and Cook criterion postulates failure whenever:

$$q = \sigma_{ci} + \frac{B}{\sqrt{3}}(3p - \sigma_{ci}) + \frac{C}{3\sqrt{3}}(9p^2 - \sigma_{ci}^2), \tag{12}$$

where p and q are the Roscoe invariants as defined in Section 2.1. The adaption of this extended Drucker–Prager criterion to the Wiebols and Cook criterion is accomplished by setting:

$$B = \frac{\sqrt{3}(n-1)}{n+2} - \frac{C}{3}(2\sigma_{ci} + (n+2)\sigma_3) \tag{13}$$

and

$$C = \frac{\sqrt{27}}{2m + (n-1)\sigma_3 - \sigma_{ci}} \times \left(\frac{m + (n-1)\sigma_3 - \sigma_{ci}}{2m + (2n+1)\sigma_3 - \sigma_{ci}} - \frac{n-1}{n+2}\right), \tag{14}$$

where $n = \tan^2(\pi/4 + \varphi/2)$ and $m = (1 + 3/5\tan\varphi)\sigma_{ci}$.

2.5. The Mogi criterion

Mogi proposed two functional relationships for rock failure, of which only the latter (Mogi 1971 criterion [21]) is considered here. In this, Mogi relates the octahedral shear stress at failure to the sum of the minimum and maximum principal stresses:

$$\tau_{oct} = f\left(\frac{\sigma_1 + \sigma_3}{2}\right), \tag{15}$$

where f is a monotonically increasing function. Plotting τ_{oct} against $\sigma_{m,2} = (\sigma_1 + \sigma_3)/2$ for different experimental data reveals that a linear function f readily gives satisfactory results, e.g. [2]. The linear Mogi criterion can be written as:

$$\tau_{oct} = a + b\left(\frac{\sigma_1 + \sigma_3}{2}\right). \tag{16}$$

Considering that in triaxial conditions $q = \sigma_1 - \sigma_3$ and that $\tau_{oct} = \sqrt{2}/3q$ (see Eq. (2)), the linear Mogi parameters a and b relate to the Coulomb shear strength parameters c and φ in triaxial compression and extension as follows:

$$a = \frac{2\sqrt{2}}{3}c\cos\varphi \quad \text{and} \quad b = \frac{2\sqrt{2}}{3}\sin\varphi. \tag{17}$$

This can be proven by substituting Eq. (16) into Eq. (4). As a result, the linear Mogi criterion is also sometimes referred to as the Mogi–Coulomb criterion [2].

2.6. The Hoek–Brown criterion

At failure, the generalized HB criterion [3] relates the maximum effective stress, σ_1 to the minimum effective