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A Lagrangian dynamic analysis of end effects in a generalized shear experiment

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Abstract

In laboratory shear testing, a primary source of error is the surcharge force caused by relative motion between the displacement actuator and dilating top sample. This force is referred to as end friction, and the changes it produces in experimental data are termed end effects. The results from a laboratory shear setup always represent a superposition of natural sliding behavior and testing machine interference; their relative proportions can be determined by externally modeling the experiment. In this paper, we construct a full Lagrangian dynamic model for the shearing behavior of a prismatic aluminum top-block over a matching asymmetric foundation. End friction is initially included in the analysis, whose viability is established by comparing modeled and experimental top-block sliding paths at 12 different shear loads. The end friction force is ultimately removed from the formulation, and the end effects manifested as the subsequent differences between modeled and experimental sliding paths. It is shown that end effects significantly alter both the sliding path and rotation mode of the prismatic top-sample. While their impact on the trajectory of a given sample appears to decrease with increasing shear force, it is shown that uniform sample scaling does nothing to alleviate the problem, and that end effects are functionally scale independent.

Keywords: Contact point; End effect; End friction; In situ behavior; Laboratory behavior; Lagrangian analysis; Lateral dilation; Sample scaling; Shear test; Sliding path

1. Introduction

The physical interaction between a testing machine and test sample inevitably produces an additional response in the sample that would not appear in situ. We term this response an ‘‘end effect’’, and its origin can be traced to the differential motion of contacting parts. In the well-known uniaxial test, for example, an end effect arises when the load platens and test sample deform laterally at unequal rates, resulting in friction along the ends of the sample. The distorting impact of end friction on uniaxial test data has been previously documented [1,2].

Whereas a uniaxial test provides information on the bulk properties of a medium, the direct shear test is used to investigate the strength of existing planar discontinuities [3]. The shear strength of a joint is usually lower than host rock compressive strength, and the movable half-sample expected to slide before material failure occurs. Because rupture or crushing are unlikely, the shear displacement force may be imparted by a point loader instead of a load platen. The point loader maintains contact with only a limited portion of the test sample and its lateral deformation does not adversely affect shear behavior. However, the roughness of a joint surface virtually ensures that a sliding sample will experience dilation, which quickly leads to relative motion between the sample and load applicator. Since relative motion is necessarily accompanied by friction, this is the manner in which end effects are introduced into a direct shear experiment.

Consider a symmetric top sample that dilates only *vertically* (Fig. 1a). According to the Mohr–Coulomb friction model, a joint with net friction angle ϕ and normal compression N requires a shear force of magnitude $N \tan \phi$ to displace. Assume that a force of such magnitude is imparted by a point loader, and define ψ as the friction angle

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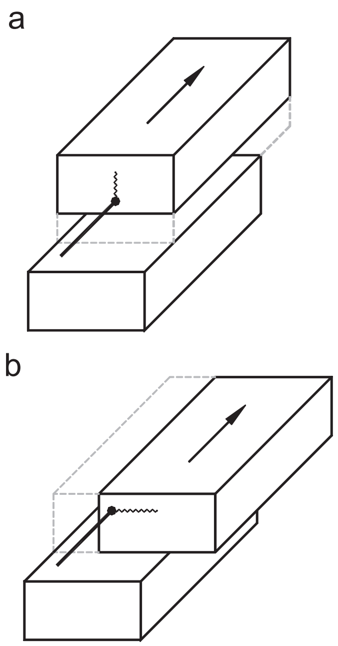

Fig. 1. Two limiting modes of dilation exhibited by a sample whose nominal displacement direction is indicated by the arrow: (a) pure vertical dilation—the sample moves upward as it is pushed forward; (b) pure lateral dilation—the sample moves sideways as it is pushed forward. The relative slip of the load point is indicated by a wavy line.

of the load tip/sample interface. The "normal" force acting on the load tip/sample interface is identical to the machine imparted displacement force, so by Mohr–Coulomb, the load tip friction force has magnitude $N\tan\phi\tan\psi$. This force acts downward as the sliding sample dilates upward and therefore supplements the total normal force. But since net compressive load $N(1+\tan\phi\tan\psi)$ is only slightly greater than command normal load N, sample dilation is unlikely to be impacted, unless we are operating near the compressive strength of the rock.

Next consider a sample which dilates only *laterally* (Fig. 1b). As before, the command normal load is N, the shear load $N\tan\phi$, and the magnitude of the load tip friction force $N\tan\phi\tan\psi$. The friction force is now directed horizontally, since relative motion between the load tip and laterally dilating sample amounts to sideslip. Because there are no externally applied lateral forces, the load tip friction force comprises the *net* external force in the horizontal direction. So whereas the vertical load of the vertically dilating sample was increased from N to $N(1+\tan\phi\tan\psi)$, the lateral load of a laterally dilating sample increases from 0 to $N\tan\phi\tan\psi$. This represents a much more serious violation of the model assumptions.

An actual joint sample will typically exhibit some combination of vertical and lateral dilation. The vertical dilation of a shearing top specimen provides information on the amplitude of the joint roughness; the tendency of a specimen to dilate laterally indicates that this roughness is not uniformly symmetric. That a rock foundation with strongly asymmetric topography can be expected to favor movements involving lateral slip is of considerable interest in dam engineering. The tendency of sliding monoliths to dilate outward as they move forward may be key to understanding the failure mechanics of a gravity dam foundation [4]. It is therefore important, when running a shear test designed to simulate the hydrostatic loading of a dam monolith, that the recorded lateral slip be a property of the foundation and not of the testing apparatus.

In this paper, we quantify the total effect of end friction on a laterally dilating tri-planar sample pair in *generalized shear*. The lower and upper half-samples under consideration are shown in Figs. 2 and 3. The experimental boundary conditions are referred to as *generalized shear* for the following reasons: (1) An asymmetric three-plane sample/sample interface produces both vertical and lateral dilation; (2) The initially matching top and bottom blocks revert to three-point contact when sliding begins (this allows the block interaction to be modeled in terms of forces rather than stresses); (3) The configuration reduces to a standard direct shear setup when the dip angles of the interface planes are set to zero; (4) The configuration extends to a more complicated discretized surface when the factual contact points are considered to be one entry in a sequence of tri-planar contacts.

A set of experimental data is first generated from shear tests performed on an aluminum sample pair. We then model the experiment mathematically using a Lagrangian (configuration space) dynamic analysis, which captures

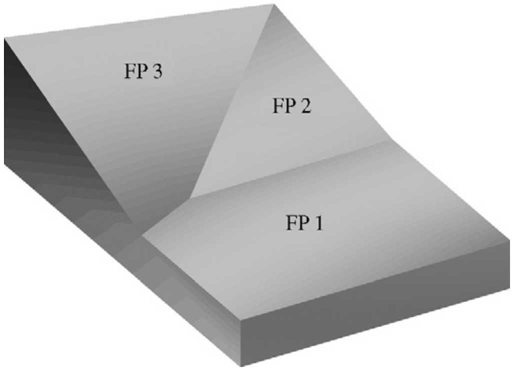

Fig. 2. Bottom sample of the shearing test pair, with foundation planes numbered.

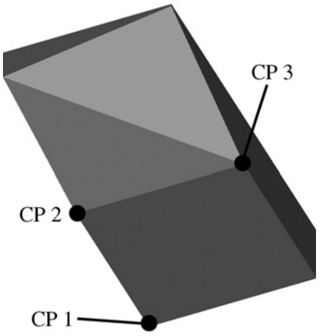

Fig. 3. Top sample of the shearing test pair, with contact points labeled.