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Application of critical plane approach to the prediction of strength anisotropy in transversely isotropic rock masses

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Abstract

A formulation describing the strength anisotropy of transversely isotropic rock masses subjected to a three-dimensional stress state is proposed based on the critical plane approach. It is assumed that the initiation of cracking is governed by the Hoek–Brown failure criterion, and the anisotropy of the strength is described through the orientation dependence of the strength parameters *m* and *s*. Using direct optimization of failure function, the direction of potential failure plane, on which the failure function reaches maximum, is determined. True triaxial compression tests as well as conventional triaxial tests are simulated in order to verify the performance of the proposed formulation.

Keywords: Strength anisotropy; Transversely isotropic rock mass; Critical plane approach; Hoek–Brown criterion; Optimization

1. Introduction

The existing experimental evidence [1–6] indicates that most sedimentary and metamorphic rocks, such as shales and slates, display a strong anisotropy of strength. These types of rocks usually exhibit some preferred orientation of fabric or possess distinct bedding planes, which results in transversely isotropic behavior on the macro-scale.

Although many attempts have been made in the past to describe the strength anisotropy of transversely isotropic rocks, no general methodology has emerged yet. The first attempt seems to be Jaeger's single weakness plane theory [7], where two independent failure modes, i.e., failure along the discontinuity and failure through intact material, were assumed to exist. The idealized distribution of triaxial strength predicted by Jaeger's theory is similar to that of curve A in Fig. 1. Here, the inclination angle β is the angle between the direction of minor principal stress and the plane of weakness. For rocks displaying a discrete fabric (i.e., multiple weakness planes), the experimental results show that the strength varies continuously with β (see curve B in Fig. 1). In order to reproduce the gradual variation of the strength, Jaeger [7] postulated that the cohesion of rock material, within the plane inclined with respect to the weakness plane, was not constant but variable depending on the angle of inclination, whereas the friction angle was considered as constant. More recently, Hoek and Brown [8] assumed that the strength parameters *m* and *s* in their well-known failure criterion are not constant but variables depending on the direction of weakness plane. However, although the values of *m* and *s* are selected based on the orientation of joints, it should be noted that the formulation still remain isotropic, so that it is doubtful whether the orientation of failure plane predicted by this approach is realistic. Another drawback of this approach, as well as the earlier one by Jaeger [7], is the requirement that the dip direction of joints should coincide with the direction of minor principal stress. In general, however, Jaeger [7] and Hoek and Brown's works [8] are of importance in that they showed that the failure criterion can be modified to take into account the anisotropy in strength properties. While the applicability of Hoek and Brown (H–B) approach is restricted, Nova [10] extended the discussion on the

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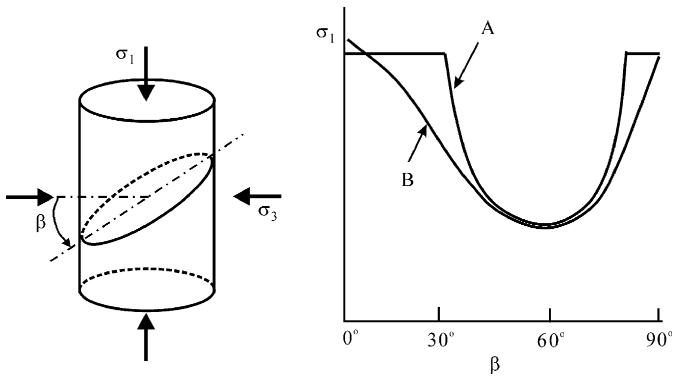

Fig. 1. Strength anisotropy in transversely isotropic rock.

anisotropy to the true triaxial stress conditions. Amadei and Savage [11] also analyzed the anisotropic strength of jointed rock having a single set of joints in three-dimensional (3D) conditions. In that work, the intact rock strength is described by the H–B criterion, whereas the joint strength is modeled by the Coulomb criterion with zero cohesion. Although the variation of material properties with orientation was not directly considered, the authors showed that the strength of the jointed rock depends on the direction of joints and the intermediate principal stress. A comprehensive review of the classical failure criteria for anisotropic rocks is provided in [9], where the authors compared the strength predictions based on different formulations.

Recently another approach, referred to as critical plane approach (CPA), has been proposed for the description of strength characteristics of geological materials [12]. This approach has been successfully applied to the analysis of sedimentary rocks [13] as well as masonry structures [14]. The methodology involves searching for a direction of failure plane, on which the value of failure function reaches maximum. The formulation incorporates the spatial variation of strength parameters whose description involves traceless second order tensors. The main advantage of CPA lies in the fact that it is formulated in general stress space and the parameters of anisotropy are described with respect to the principal material triad, so that the numerical implementation becomes systematic and straightforward. On the other hand, its disadvantage is the additional calculation time that is required to establish the direction of critical plane by a suitable optimization scheme. Also, the predictive abilities of the framework are affected by the layering. In general, the performance of CPA is better for periodic microstructures, such as rock systems with equally spaced homogeneous layers.

In this paper, CPA is applied to describe the strength properties of transversely isotropic H–B rock masses. The anisotropy is defined by postulating that the H–B strength parameters m and s are exponential scalar-valued functions of the orientation in space. The optimization scheme employed to find the critical direction is briefly discussed. The paper is concluded by presenting some numerical examples which illustrate the performance of the proposed model.

2. Failure function

H–B failure criterion is expressed as the following empirical nonlinear relation:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}, \tag{1}$$

where σ_1 is the major principal stress at failure, σ_3 is the corresponding minor principal stress, σ_c is the uniaxial compressive strength of intact rock material, and m and s are strength parameters characterizing the rock mass considered.

The local form of this criterion, in terms of traction components on a plane, has been provided in the article by Hoek [15]. It is defined by the following set of parametric equations:

$$\tau_f = (m\sigma_c/8)f(\sigma_f), \tag{2a}$$

$$f(\sigma_f) = \cot\phi - \cos\phi, \tag{2b}$$

$$\phi = \tan^{-1}(1/\sqrt{4h\cos^2\delta - 1}), \tag{2c}$$

$$3\delta = 90 + \tan^{-1}(1/\sqrt{h^3 - 1}), \tag{2d}$$

$$h = 1 + 16(m\sigma_f + s\sigma_c)/(3m^2\sigma_c), \tag{2e}$$

where τ_f and σ_f are the shear and normal stresses acting on a failure plane, and the compressive stress is considered as positive. Eq. (2c) gives the instantaneous friction angle, so that the corresponding instantaneous cohesion, c, can be obtained from the relation

$$c = \tau_f - \sigma_f \tan\phi. \tag{3}$$

Hence, the failure function for a H–B material, defined in $\sigma - \tau$ plane, takes the form

$$F = \tau - (m\sigma_c/8)f(\sigma). \tag{4}$$

3. Definition of anisotropy in strength parameters

It is assumed that, on a plane having the unit normal $\mathbf{n}$, the strength parameters m and s appearing in Eq. (1) can be defined in terms of the following distribution functions:

$$m = a_1^m + a_2^m \exp(\mathbf{n}\cdot\mathbf{\Omega}^m\mathbf{n}), \tag{5a}$$

$$s = a_1^s + a_2^s \exp(\mathbf{n}\cdot\mathbf{\Omega}^s\mathbf{n}), \tag{5b}$$

in which $\mathbf{\Omega}$'s are the second order tensors which describe the bias in the spatial distribution of strength parameters, whereas $a_{1,2}^m$ and $a_{1,2}^s$ are coefficients that are independent of direction. Furthermore, the $\mathbf{\Omega}$'s are symmetric traceless tensors whose principal directions coincide with the material axes [8]. It should be noted that for an isotropic material the Ω's vanish, so that m and s become constant.