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Research paper

Control of tension–compression asymmetry in Ogden hyperelasticity with application to soft tissue modelling

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ABSTRACT

This paper discusses tension–compression asymmetry properties of Ogden hyperelastic formulations. It is shown that if all negative or all positive Ogden coefficients are used, tension–compression asymmetry occurs the degree of which cannot be separately controlled from the degree of non-linearity. A simple hybrid form is therefore proposed providing separate control over the tension–compression asymmetry. It is demonstrated how this form relates to a newly introduced generalised strain tensor class which encompasses both the tension–compression asymmetric Seth–Hill strain class and the tension–compression symmetric Bažant strain class. If the control parameter is set to $q=0.5$ a tension–compression symmetric form involving Bažant strains is obtained with the property $\Psi(\lambda_1,\lambda_2,\lambda_3)=\Psi\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)$. The symmetric form may be desirable for the definition of ground matrix contributions in soft tissue modelling allowing all deviation from the symmetry to stem solely from fibrous reinforcement. Such an application is also presented demonstrating the use of the proposed formulation in the modelling of the non-linear elastic and transversely isotropic behaviour of skeletal muscle tissue in compression (the model implementation and fitting procedure have been made freely available). The presented hyperelastic formulations may aid researchers in independently controlling the degree of tension–compression asymmetry from the degree of non-linearity, and in the case of anisotropic materials may assist in determining the role played by, either the ground matrix, or the fibrous reinforcing structures, in generating asymmetry.

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1. Introduction

Realistic constitutive modelling for biological soft tissue is relevant to research areas such as impact biomechanics (Ivancic et al., 2007; Muggenthaler et al., 2008; Forbes et al., 2005), rehabilitation engineering (Linder-Ganz et al., 2007, 2008; Portnoy et al., 2008, 2009), tissue engineering (Hinds et al., 2011), gait analysis (Mathur et al., 2010), surgical simulation (Lim et al., 2007; Audette et al., 2004; Famaey and Vander Sloten, 2008), and modelling of soft tissue drug transport (Wu and Edelman, 2008; Wu et al., 2009). Many biological materials present with significantly different behaviour for tensile or compressive loading (e.g. muscle tissue (Gindre et al., 2013), cervical tissue (Myers et al., 2015), bone (Niebur et al., 2000), intervertebral disk (Cortes and Elliott, 2012), and cartilage (Nagel and Kelly, 2010). This is known as tension–compression asymmetry. Further, biological materials are also non-linear elastic, and often anisotropic due to the presence of fibrous connective tissue structures. Anisotropy can be modelled by combining an isotropic ground matrix with fibrous reinforcement. Through adjustment of material parameters current hyperelastic constitutive formulations offer control over the dominance of either the ground-matrix or the fibrous components, as well as the degree of non-linearity in their response. Tension–compression asymmetry can be present in the behaviour of isotropic formulations, and therefore ground-matrix formulations, as well as in fibrous reinforcement components. However, in many formulations the constitutive parameters dictating the degree of non-linearity also affect the degree of tension–compression asymmetry. As such the source and degree of tension–compression asymmetry is often not specifically controlled in the constitutive formulation. This study presents constitutive formulations, based on Ogden hyperelasticity, for isotropic materials, such as ground-matrices, offering separate control over the degree of non-linearity and tension–compression asymmetry. Such formulations may aid in the identification of the role played by either the ground matrix, or the fibrous reinforcing structures, in generating tension–compression asymmetry.

According to the representation theorems in non-linear continuum mechanics (see also Holzapfel, 2000; Bonet and Wood, 2008; Ogden, 1984) for isotropic materials the strain energy density function defining constitutive behaviour for finite elasticity can be formulated in terms of principal invariants $\{I_1, I_2, I_3\}$ or principal stretches $\{\lambda_1, \lambda_2, \lambda_3\}$ such that $\Psi(I_1, I_2, I_3) = \Psi(\lambda_1, \lambda_2, \lambda_3)$. Several successful formulations have been proposed for incompressible rubber-like materials (see also Treloar et al., 1976). A general first order expression for incompressible materials in terms of principal invariants is given by the so called Mooney–Rivlin hyperelastic model (Mooney, 1940; Rivlin, 1948a, 1948b) often presented as:

$$\Psi(I_1(\mathbf{C}), I_2(\mathbf{C})) = C_1(I_1(\mathbf{C}) - 3) + C_2(I_2(\mathbf{C}) - 3) \tag{1}$$

For incompressible materials, $J = \det(\mathbf{F}) = 1$ and therefore $I_2(\mathbf{C}) = I_1(\mathbf{C}^{-1})$, allowing the Mooney–Rivlin form to be rewritten in terms of principal stretches as:

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = C_1\left({\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^2 - 3\right) + C_2\left({\lambda_1}^{-2} + {\lambda_2}^{-2} + {\lambda_3}^{-2} - 3\right) \tag{2}$$

Mooney (1940) derived this form by postulating isotropy, incompressibility, and the requirement that tractions for simple shear are proportional to the shear. To capture non-linear behaviour for finite deformations Mooney (1940) proposed a more general form whereby tractions were postulated to be explicit functions of the shear:

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_{m=1}^{\infty}\left[A_{2m}\left({\lambda_1}^{2m} + {\lambda_2}^{2m} + {\lambda_3}^{2m} - 3\right) + B_{2m}\left({\lambda_1}^{-2m} + {\lambda_2}^{-2m} + {\lambda_3}^{-2m} - 3\right)\right] \tag{3}$$

(follows original notation by Mooney (1940), note that $m \in \mathbb{N}$, and plays the role of a subscript index, for the constitutive parameters, and appears in the exponent for the stretches). It can be seen that if $A_{2m} = B_{2m}$ this form has the tension and compression symmetry property $\Psi(\lambda_1, \lambda_2, \lambda_3) = \Psi\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}\right)$. Mooney (1940) and more specifically Rivlin (1948a and 1948b) argued the strain energy density should be a symmetrical and even-powered function of the principal stretches. Mooney (1940) also presented a form offering control over asymmetry by using $A_{2m} = \frac{G_{2m} + H_{2m}}{4}$ and $B_{2m} = \frac{G_{2m} - H_{2m}}{4}$:

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_{m=1}^{\infty}\left[\sum_{i=1}^{3}\frac{G_{2m}}{4m}\left({\lambda_i}^{2m} + {\lambda_i}^{-2m} - 2\right) + \sum_{i=1}^{3}\frac{H_{2m}}{4m}\left({\lambda_i}^{2m} - {\lambda_i}^{-2m}\right)\right] \tag{4}$$

Ogden (1972a and 1972b) also removed the symmetry constraint, and, since stretches are naturally positive quantities, dropped the requirement for integer and even-powers leading to the highly flexible form:

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_{a=1}^{N}\frac{c_a}{m_a}\left({\lambda_1}^{m_a} + {\lambda_2}^{m_a} + {\lambda_3}^{m_a} - 3\right) = \sum_{a=1}^{N}\left[\frac{c_a}{m_a}\sum_{i=1}^{3}\left({\lambda_i}^{m_a} - 1\right)\right]$$

With $m_a \in \mathbb{R}$, and $(c_a m_a) \in \mathbb{R}_{>0}$ (5)

The Ogden hyperelastic formulation has been employed to a great extent for incompressible rubber-like materials (Ogden, 1986; Marckmann and Verron, 2006) and has recently been shown to agree with the statistical theory of microscopic fibre networks (Ehret, 2015). Typically for rubber-like materials parameter fitting of the Ogden form involves 3–4 terms, whereby 1 term involves negative m_a and 2–3 terms involve positive m_a values (see also Marckmann and Verron, 2006; Ehret, 2015; Ogden et al., 2004). Since mechanical testing of biological samples is more challenging, and the data is often of a sparser nature compared to data for engineering materials, reduced order models are often employed leaving fewer parameters to be identified. For instance, 1st order Ogden formulations have been used for skeletal muscle tissue (Bosboom et al., 2001) and skin (Groves et al., 2012). In this case only positive m_a values are used. For such reduced order formulations, as will be demonstrated in this paper, a tension–compression asymmetry exists. When the parameters controlling the degree of non-linearity (the m_a values) are adjusted the asymmetry is also affected. Hence