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Research Paper

# Reliability of structure tensors in representing soft tissues structure

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ABSTRACT

Structure tensors have been applied as descriptors of tissue morphology for constitutive modeling. Here the reliability of these tensors in representing tissues structure is investigated by model simulations of a few examples of generated and measured planar fiber orientation distributions. Reliability was evaluated by comparing the data with the orientation density distribution recovered from the structure tensor representation, and with a orientation density recovered from an alternative representation by Fourier series of spherical harmonics. The results show that except for the case of uniform or close to uniform orientation distributions, the distributions recovered from the structure tensor fit the data poorly. On the other hand, orientation distributions recovered from Fourier series of spherical harmonics converge to the data distributions provided sufficient terms are included in the truncated series. These results suggest that the structure tensor is a reliable descriptor of tissue structure only under very limited cases.



## 1. Introduction

Structural modeling of tissue mechanics incorporates as key determinants both the fibers orientation distribution and their uniaxial properties. The fibers' orientation dispersion is represented by a density distribution function (ddf) which can be expressed based on the spherical polar angles $\phi, \theta$ as $\psi(\phi, \theta)$. The orientation ddf representation has been applied for a variety of tissues such as skin, myocardium, articular cartilage, arteries, veins and aortic valves (Ateshian et al., 2009; Belkoff and Haut, 1991; Billiar and Sacks, 2000; Decraemer et al., 1980; Freed et al., 2005; Girard et al., 2009; Hollander et al., 2011b; Lanir et al., 1996; Nevo and Lanir, 1989; Nguyen et al., 2008; Pandolfi and Holzapfel, 2008; Sacks, 2003; Wuyts et al., 1995; Zulliger et al., 2004).

Constitutive characterization of tissues based on the ddf structural representation has several advantages. First, it was shown to be superior to other approaches in its descriptive (fit to experimental data) and predictive capabilities (Hollander et al., 2011a). Second, different types of fibers can be readily added by summing the effects of all fiber networks, each with its own orientation ddf and mechanical properties (Lokshin and Lanir, 2009). Third, the requirement of convexity of the constitutive formulation is automatically satisfied in the structural formulation since the mechanical responses of biological fibers are convex (Lanir, 1996). Finally, fibers' non-elastic properties can be readily incorporated into the model by replacing the fiber hyper-elastic stress with its non-elastic one, such as fiber viscoelasticity and pre-conditioning adaptation (Lokshin and Lanir, 2009; Raz and Lanir, 2009).

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Constitutive formulation based on the orientation ddf has a numerical disadvantage: it involves double integration over the entire range of the polar spherical angles $\phi, \theta$. This computational burden may impede application of structure based material laws in finite element analysis. An analytical closed form structure based model which circumvents this difficulty has been developed, but only for membrane tissues subjected to biaxial stretch where all the fibers are stretched, and for particular choices of the orientation ddf and fiber mechanical properties (Raghupathy and Barocas, 2009). Yet, as pointed out by Federico and Herzog (Federico and Herzog, 2008), if some fibers buckle (as may happen under deformation protocols such as uniaxial stretch and shear tests), then analytical derivation is not possible and numerical schemes are required.

To reduce the computational effort associated with double integration over the respective ranges of the polar angles, several studies have applied the generalized structure tensor as representation of the fibers orientation dispersion (Federico and Herzog, 2008; Freed et al., 2005; Gasser et al., 2006). It was also applied for analysis of cell traction-induced alignment of collagen fibrils and cells in a cell-seeded gel (Barocas and Tranquillo, 1997), and in analysis of remodeling induced evolution of soft tissue structure (Driessen et al., 2003). Structure tensors have been previously applied for modeling the flow of polymer melts (McLeish and Larson, 1998). In common with melts literature, the higher efficiency in using the structure tensor is obtained by applying constitutive laws for the fibers' strain energy function (and stresses) which depend only on a single global measure of the fibers' deformation rather than on the actual deformation which is orientation dependent. This single measure is derived from both the tissue global strain and the structure tensor. An example of such a measure commonly used is the weighted orientation-averaged squared stretch $\overline{\lambda}^2$.

A question which naturally arises regarding the application of structure tensor relates to its reliability in representing the tissue's structure and predicted response. There are two independent aspects involved, one relating to the bias introduced by applying a single global measure of the deformation to represent the fiber response. The other is a more fundamental one: how reliable is the structure tensor in representing the true fiber orientation distribution. The first question was previously addressed (Federico and Herzog, 2008) and numerically investigated (Cortes et, al. 2010) by comparing predictions of the main normal and shear stress components, between the full structural characterization and the structure tensor one, under uniaxial, biaxial and shear test protocols. The results showed that both formulations are equivalent only for limited cases of planar distributions under equibiaxial stretch, and when the fiber dispersion is very small. Federico and Herzog (Federico and Herzog, 2008) pointed out that the good fit using this approach for blood vessels (Gasser et al., 2006) was obtained since the fibers orientation dispersion was small, combined with the fact that all fibers were loaded in tension. However, under a general deformation scheme (e.g., uniaxial stretch, shear) some fibers may contract and buckle thereby biasing the calculated $\overline{\lambda}^2$ in a non-physical manner. To deal with this bias, they proposed to exclude contracted fibers when calculating $\overline{\lambda}^2$.[1]

The aim of this communication is to address the second concern relating to the reliability of the structure tensor descriptor of the tissue fibers' orientation distribution.

## 2. Methods

### 2.1. Mathematical background

#### 2.1.1. Structural tissue characterization

In the structural approach to tissue characterization (Lanir, 1979, 1983) the fibers' orientation dispersion is quantified by a density distribution function of the polar angles, $\psi(\phi,\theta)$. Being a density distribution (ddf), $\psi(\phi,\theta)$ must obey the normalization condition

$$\int_\phi \int_\theta \psi(\phi,\theta) \cdot \sin\phi \cdot d\phi \cdot d\theta = 1 \tag{1.1}$$

In addition, $\psi(\phi,\theta)$ is even, i.e., a fiber oriented at any angle $(\phi,\theta)$ is indistinguishable from a fiber oriented in the opposite direction at an angle $(\pi-\phi, \theta+\pi)$.

For tissues with hyperelastic fibers, their strain energy function equals the weighted sum of the fiber strain energies. This is thus expressed by[2]

$$W_f(\mathbf{C}) = \phi_f^0 \int_\phi \int_\theta \psi(\phi,\theta) \cdot w_f(\lambda_f) \cdot \sin\phi \cdot d\phi \cdot d\theta \tag{1.2}$$

The fiber network second Piola–Kirchoff stress is derived from (1.2) by chain differentiation,[3]

$$\mathbf{S}_f(\mathbf{C}) = 2 \cdot dW_f/d\mathbf{C} = \phi_f^0 \int_\phi \int_\theta \psi(\phi,\theta) \cdot s_f(\lambda_f) \cdot \mathbf{NN} \cdot \sin\phi \cdot d\phi \cdot d\theta,$$
$$\lambda_f^2 = \mathbf{N}^T\mathbf{C}\mathbf{N}, \quad \mathbf{N}(\phi,\theta) = \sin\phi \cdot \cos\theta \cdot \mathbf{e}_1 + \sin\phi \cdot \sin\theta \cdot \mathbf{e}_2 + \cos\phi \cdot \mathbf{e}_3, \tag{1.3}$$

where $\mathbf{C}$ is the right Cauchy–Green strain tensor, $\phi_f^0$ is the reference fibers volume fraction, and $\mathbf{e}_i$ are unit base vectors in a Cartesian coordinate system. The functions $w_f(\lambda_f)$ and $s_f(\lambda_f) = (2/\lambda_f) \cdot dw_f/d\lambda_f$ are respectively the strain energy and second Piola–Kirchoff stress of a uni-directional parallel fiber bundle as a function of its orientation dependent stretch $\lambda_f$. $\mathbf{N}$ is a unit vector in the bundle reference orientation. Eq. (1.3) results from the assumed affine deformation and $\mathbf{NN} = (d\lambda_f/d\mathbf{C})/2$ is the direct vector product of $\mathbf{N}$ with itself.

#### 2.1.2. The structure tensor method

The structure tensor $\mathbf{H}$ applied in previous studies is related to the orientation ddf $\psi(\phi,\theta)$ by

[1]*Actually, it is easy to show that application of $\overline{\lambda}^2$ may lead to absurd results. Consider a simple case of an isotropic incompressible tissue with* *(footnote continued)* *fibers equally distributed between three normal directions parallel to the Cartesian coordinates. Under uniaxial stretch, fibers in the lateral directions will contract and buckle. In contrast, the structure tensor method with $\overline{\lambda}^2$ as a deformation measure predicts these fibers to be stretched. Federico and Herzog (Federico and Herzog 2008) proposal to exclude compressed fibers from $\overline{\lambda}^2$ evaluation does not remedy this problem.*

[2]The term $\sin\phi$ *in the integrands of (1.1), (1.2), (1.3), comes from the Jacobian of the polar spherical coordinate system. In some earlier studies it was incorporated into $\psi(\phi,\theta)$.*

[3]*In cases of non-elastic fibers the hyper-elastic fiber stress $s_f(\lambda_f)$ is replaced by its non-elastic counterpart $s_f(\lambda_f, t)$.*