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Appraisal of Strouhal number in wind turbine engineering

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ABSTRACT

Flows around blunt bodies at high Reynolds numbers generate a periodic release of staggered vortices. The dimensionless frequency of vortex shedding, the Strouhal number *St*, was found successful in describing periodic fluid flows. In fact, several literature datasets proved that the number $St \sim 0.16$ describes with reasonable (at times, excellent) approximation a large variety of periodic fluid phenomena, many of them having no clear affinity. The first motivation of the present study is to collect under one cover the results disseminated in various sources; the second purpose is to double-check the constancy of Strouhal number in wind turbines as well. The *St* number is here elaborated in a more informative way for horizontal-axis wind turbines, showing that it includes the tip speed ratio and the number of blades; the law $St \sim 0.16$ is here corroborated by further findings which includes wake meandering and the flow fields produced by three wind turbines (a two-bladed HAWT; a three-bladed lift-driven Darrieus VAWT; and a two-bladed drag-driven Savonius VAWT). The present findings suggest a design criterion for wind turbines.

Contents

Abbreviation: C_{FL}, Courant–Friedrichs–Lewy number; GF, growth factor; HAWT, horizontal-axis wind turbine; VAWT, vertical-axis wind turbine

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List of main symbols

c	airfoil chord (L)
$D=2R$	rotor diameter (also width scale) (L)
f	frequency (T^{-1})
L	total distance of influence (L)
N	number of blades (–)
$St=f\ell/V$	Strouhal number (–)
V_∞	undisturbed freestream wind velocity (LT^{-1}).
$\lambda=\omega R/V_\infty$	rotor tip speed ratio (–)
$\Lambda=\omega R_g/V_\infty$	tip speed ratio of the rotating grid (–)
π	3.14159… (–)
ω	blade angular velocity (T^{-1})

1. Introduction

1.1. Tip speed ratio and Strouhal number

The tip speed ratio λ (also known as *TSR*) is one of the most noteworthy parameters in wind engineering. It is defined as the ratio between the maximum tangential velocity (e.g., at the tip of the revolving blade) and the reference velocity:

$$\lambda=\frac{\omega R}{V_\infty}, \tag{1}$$

where ω is the angular velocity of the blade; R is the rotor radius and V_∞ is the undisturbed freestream velocity of the wind.

The Strouhal number, *St*, is defined as

$$St=\frac{f\ell}{V}, \tag{2}$$

where f, ℓ and V are respectively the appropriate frequency, length and velocity scales which can best capture the unsteady nature of the problem under investigation. The Strouhal number was conceived to tackle periodic fluid flows.

1.2. Literature evidences of the Strouhal number

The knowledge of shedding frequencies is useful in many applications that have been exploited in the literature (such as, to name a few, shedding behind electrical and cable-way wires; chimneys; submarine conning towers; aeroelastic resonance in bridges) and the purpose of this study is to show possible applications of the *St* law also in wind turbines.

Flows around blunt bodies at high Reynolds numbers generate a periodic release of staggered vortices having alternating circulation (the *vortex street*); these coherent vortex structures are apparently related to fluctuations of drag and lift forces experienced by the body. One of the main features of this phenomenon is the similarity of the vortex street which, in fact, is weakly dependent on the geometry of blunt bodies. This fact suggested a possible constant relation between the Reynolds number (which takes into account the transversal scale length of the body) and the parameter describing flow unsteadiness, such as the Strouhal number.

The first value $St=0.185$ was assessed in 1878 by Strouhal [1]. Roshko [2] observed similarity among vortex streets downstream of cylinders of different cross sections and postulated $St\sim0.16$ for the periodic shedding past 2*D* blunt bodies. For the wake of cylinders of high aspect ratio, Steinman [3] reported $St\sim0.18$ in the Reynolds range $10^3<Re<10^5$. Castro [4] measured shedding frequencies behind perforated plates in the range $2.5\cdot10^4<Re<9\cdot10^4$ where the measured *St* number (based on the plate width) reaches asymptotically the value $St\sim0.15$. For turbulent 2D flapping jets, Cervantes and Goldschmidt [5] measured $St\sim0.154$. For square cylinders of aspect ratio 3 and symmetrically confined within solid walls with low blockage effects, Okajima [6] found $St\sim0.16$. A common *St* number, although defined in a different way, was also recognized by Bejan [7] in case of jets and wakes; later on, Cervantes et al. [8] demonstrated that the *St* formulation conceived by Bejan [7] is identical to the *St* definition (2), apart from a multiplying constant.

Bearman [9] measured $St=0.19$ and $St=0.46$ respectively before $(Re=2\cdot10^5)$ and after $(Re=4.1\cdot10^5)$ the drag crisis of circular cylinders. Griffin [10] measured similarity in wakes of steady/vibrating blunt bodies and proposed a variant of the *St* number. Adachi et al. [11] compared the *St* variants proposed by Roshko [2], Bearman [9] and Griffin [10], finding that Bearman [9]'s approach prompts $St\sim0.18$ at high *Re* numbers.

The first attempt to attain a comprehensive understanding of this issue was carried out by Levi [12], who conceived a simple and effective mechanical model based on the assumption that an oscillating fluid body of width scale *D*, excited by an outer flow V_∞, is represented by a harmonic oscillator vibrating with frequency f according to a resonator model with characteristic vibration energy $0.5(2\pi fD)^2$. Equating the vibration energy with the specific kinetic energy of the flow, $V_\infty^2/2$, yields

$$St=\frac{1}{2\pi}\sim0.16. \tag{3}$$

Levi [12] collected plenty of evidences to prove the accuracy of relation (3) in a large variety of fluid flows [13–22]. Relation (3) proved correct even when applied in a geophysical context (such as estuary flows [23]) or in wall bounded turbulence (e.g. turbulent bursts [24]). The main outcome of Levi [12]'s approach is to show that Roshko [2]'s law is more general and valuable than it was initially expected.

Dominguez and Peralta-Fabi [25] provided an additional example of Levi [12]'s model in case of vortices shedding behind a sphere confined in a cylindrical conduit. Ahlborn et al. [26] proposed a phenomenological model capable of embodying Roshko [2]'s results. In wind tunnel experiments, Medici and Alfredsson [29] measured $0.12<St<0.20$ for the large-scale vortices downstream a two-bladed HAWT. Cigada et al. [36] and Malavasi and Zappa [37] measured $0.12<St<0.17$ for vortices shedding of square cylinders of different aspect ratios. A constant Strouhal number was amazingly found even for breaking periods of 2D bores [38]; the value $St=0.22$ was discovered by accurate numerical simulations and a clever analogy (Fig. 12 of [38]) was conceived with the circular cylinder.

Morton [39] mentioned the studies [40–42] dealing with flat plates of different aspect ratios at various *Re* numbers, in which $0.14<St<0.17$. Morton [39] cited [43–46] for cylinders, wedges and V-gutter, finding $0.143<St<0.245$. Rosetti et al. [47] found by *unsteady* numerical computations $St\sim0.24$ for smooth circular cylinders at high *Re* numbers. Chein and Chund [48] measured $0.14<St<0.172$ for a flat plate at different inclinations.

As for two-phase phenomena, Bakić and Perić [49] measured $St\sim0.2$ for unsteady cavitation in the wake of a sphere; Brandner et al. [50] also found $0.1<St<0.2$ for a very large interval of cavitation stages (cavitation numbers comprised between 0.90 and 0.40).

Applications of Strouhal numbers can be even found in animal biomechanics, where propulsive efficiency of flying and swimming animals has also been interpreted in terms of Strouhal number; it has been observed that dolphins and sharks swim mostly in the